

UNIVERSITY OF CALIFORNIA  
SANTA CRUZ

**IMPROVED FIELD EXPLORATION WITH INTERACTIVE PATH  
PLANNING**

A thesis submitted in partial satisfaction of the  
requirements for the degree of

MASTER OF SCIENCE

in

COMPUTER ENGINEERING

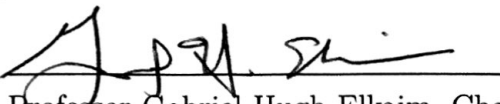
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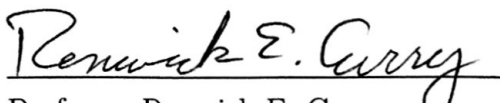
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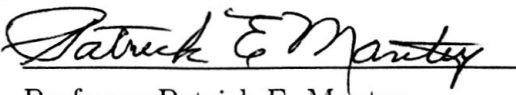
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## Abstract

### Improved Field Exploration with Variance Suppressing Path Planning

by

Sargis S Yonan

A set of methods and path planners are introduced for the exploration of unknown semi-to-fully-ergodic fields of interest. The spatial statistical properties of a target field can be exploited to assist in variance suppressing planning techniques from observations of a single state of interest. The Kriging Method, a *Best Linear Unbiased Predictor*, is used to exploit the statistical properties, namely the spatial autocorrelation, of a target field. The Kriging Method predicts the state of unobserved points from a set of observed points for the purposes of quality mapping. A prediction and confidence of prediction of the entirety of a given target field can be generated from the method.

The path planners introduced can be used to reduce the overall prediction uncertainty of a field by steering a single vehicle to collect a good set of samples. A metric for return on investment of executing a trajectory using feedback from Kriging predictions is presented. The five path planners introduced suppress the overall uncertainty of a Kriging prediction of an unknown target field in order to create a higher quality map when compared to a preplanned scanning regime, and another Kriging variance suppressing method (Greedy Next-Best-View), for the same distance traveled.

For my mother, Marina

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# Chapter 1

## Introduction

Field exploration is a method in which an unknown field (a *target field*) is learned in an attempt to discover traits or track trends about the field. Field exploration methods can be useful for tracking the health of crop soil, the size of ice glaciers, generating terrain maps, and a wide variety of scientific, agricultural, and industrial purposes. Furthermore, an exploration technique, versus a patrolling or tracking technique where a target is tracked or surveilled, does not require a model of the target field dynamics, as they can be learned on-the-fly.

Using an autonomous exploration vehicle, an unknown field of interest can be scanned within a more reasonable time frame, and cost, when compared to conventional scanning techniques involving satellite and manned-airplane missions. Using the techniques introduced, a high-quality map can be generated of a previously unknown field of interest on demand. Satellite imagery of Earth has been used for measuring various natural phenomena in the past several decades. Estimating polar ice cap melting rates and exploring the

locations of oil spills are among the class of problems solved by exploration techniques. The US Forest Services' Moderate Resolution Imaging Spectroradiometer (MODIS) Active Fire Mapping Program updates images every one to two days with a fixed sensor payload in orbit. While this program is helpful for detecting large events with long periods of activity, the sampling rate of this service might not give an emergency response team or a scientist the required resolution and precision in gathered data at their desired rate. The resolution and frequency problem along with the cost associated with building, launching, and maintaining an orbiting Earth satellite might make some areas of research prohibitive. The use of unmanned aerial vehicles (UAVs), and other autonomous vehicles, have more recently been used in similar fields of study, industry, agriculture, and in environmental protection. The benefit in using these vehicles is more rapidly acquired data with adjustable accuracy. A UAV, for example, can give more nuanced and detailed data on features of a field that are not observable from the distance or field of view of an orbiting satellite with a fixed payload. The autonomous vehicle can be equipped with any flyable or drivable sensor that can be deployed from virtually anywhere to explore anywhere within reach.

A common approach to exploring a field is to conduct a zig-zag pattern, or other pre-determined maneuver on a target field. This task might take longer than needed to collect the required data, and could potentially ineffectively use the flight or drive time of the exploration vehicle which often has a limited runtime. A method that utilizes the learned stochastic properties of a field could be used to decrease exploration time by avoiding the need to scan more points than needed. A majority of the unobserved points in a field can be predicted to a known degree of confidence, given a degree of spatial autocorrelation in



the field. Furthermore, scanning every point in a large unknown field is an unrealistic expectation for vehicles with limited maneuvering capabilities. This is especially a problem if the field as a whole is very large and needs to only be predicted to a small degree of confidence. A scheme for minimal and high-quality scanning via variance suppressing path planning would be in the benefit of time for the user(s) of such a system, and the scanning equipment as well.

Using the Kriging variances generated from a set of samples taken on a field, variance based path planning methods can be used to steer an exploration vehicle in the areas of maximal uncertainty, while traversing over areas of low prediction confidence. The methods introduced attempt to help a user of this system explore an unknown field with a known degree of confidence that is configurable through a desired runtime, tuned by the user.

## 1.1 Previous Works

The goal of this thesis is to introduce path planning techniques which reduce overall uncertainty of Kriging field predictions by steering a single vehicle through a field optimally. We use Kriging predictions as feedback into a path planner to estimate confidence return for a given candidate trajectory.

Exploration is a subset of the types of missions UAVs have been used for recently. From Section 2 of Nikhil Nigam’s *The Multiple Unmanned Air Vehicle Persistent Surveillance Problem: A Review* [16], the various types of missions possible are described. There exist problems of tracking and patrolling which involve following a moving target, or of finding the spread rate and source of an item of interest. The exploration mission type is a pro-

cedure which runs parallel to the these types of missions. Without a model describing the states of the item of interest being explored, a simple scanning procedure involving random movements or following a predetermined path, like a zig-zag about the field as in [21] are executed, or a zig-zag which incorporates the model dynamics of the vehicle, as in [15].

In *Autonomous Aeromagnetic Surveys Using a Fluxgate Magnetometer* by Douglas G. Macharet et al., A UAV is used in a mineral field exploration technique, where a fluxgate sensor is used to measure the magnetic flux of a point beneath the UAV [5]. A zig-zag pattern is ultimately used to explore the field for minerals of interest. A more dynamic strategy is used in the autonomous home vacuum cleaner *Roomba* by iRobot, where a spiral pattern is used in an attempt to clean up and find the periphery of debris [8]. The radius of the spiral pattern is a function of the amount of debris tracked by the debris sensor in the immediate area of the vacuum cleaner.

Exploration missions often do not specify the model of the item of interest being tracked. Knowing the model and kinematics of the item being tracked makes it possible to use an optimal estimation tool such as an Extended Kalman Filter as in Rabinovich et al. *A Methodology For Estimation of Ground Phenomena Propagation* [19] and *Multi-UAV Path Coordination Based on Uncertainty Estimation* [18] where the velocity and position states of a ground fire are estimated while tracking the points surrounding the periphery of a wildfire. The planner for Rabinovich’s mission calculates a path based on the Kalman variances of the control points representing the periphery of the ground phenomenon being tracked.

The Kriging Method has been used in a UAV Contour Tracking problem in Zhang et al. *Oil Spills Boundary Tracking Using Universal Kriging And Model Predictive Control By*

UAV [26]. The work relies on the knowledge of a model of the oil spill, and therefore is not a generic case of an exploration problem of a model-less field.

C. C. Castello et al. present the use of the Kriging method for environmental sensor placement in *Optimal Sensor Placement Strategy for Environmental Monitoring using Wireless Sensor Networks* [1]. The overall variances of a Kriging predicted field, predicted from a set measurements from fixed sensor locations, can be directly compared to the variances of predicting the same field with a different set of sensor locations. The method can therefore be used to help assist in optimal sensor placement by conducting a Monte Carlo simulation of random sensor placements, and ultimately choosing the random configuration that minimized the Kriging prediction variances for the field. A path planner, which can be stated as a sensor placement problem, by selecting a random path, or set of sampling locations, that minimizes the expected Kriging variance of a target field is introduced in this thesis in Section 3.5. The use of a Monte Carlo approach, where noise is used to assist in suppressing prediction uncertainty has been used for uncertainty suppression in obstacle avoidance motion planning in *Monte Carlo Motion Planning for Robot Trajectory Optimization Under Uncertainty* [12], but the technique was not used for exploration purposes, as introduced in this thesis.

Near the completion of this thesis, a paper discussing the benefits of using Kriging variance motivated path planning for field exploration, was published [17]. Pulido Fentanes et al. published *Kriging-Based Robotic Exploration for Soil Moisture Mapping Using a Cosmic-Ray Sensor*, where a Kriging variance based exploration technique is used for the purpose of quality mapping of agricultural soil moisture [17]. In the publication, a set of

Kriging path planners are used to reduce Kriging prediction error by steering an agricultural robot into areas of high Kriging variance calculated by using all data gathered on a path.

The first of their path planners, named *Greedy Next-Best-View* (NBV), similar to the *Highest Variance* (HV), Gradient Ascent (GA), and Range Gradient Ascent (RGA) strategies demonstrated in this thesis, simply targets the point of highest Kriging variance from a set of candidate locations. In the Greedy NBV algorithm, the path is recalculated every time the robot takes a sample. In HV, GA, and RGA, a path is only recalculated when the last set decision point is met. The GA path planner from this thesis further differs from Greedy NBV by stepping into the point of highest variance surrounding the exploration vehicle, and not in the direction of highest field variance. The authors of the publication also introduce a *Monte Carlo Next-Best-View* where a set of random endpoints are generated and weighted against one another according to their Kriging variances. The endpoint with the highest uncertainty is selected as the next sample location.

In this thesis, a Monte Carlo technique is also introduced in Section 3.5, but instead of weighing each proposed random trajectory by its Kriging variance, the predicted values of the points along a random trajectory are reused, and a Kriging variance calculation is run on the field again. The path that is ultimately chosen is the path which reduces the expected overall Kriging variance of the field as a whole. Lastly, the authors in [17] introduce an adaptive sampling planner which works by generating an initial path that is then modified after each sample taken. As more possible path are generated randomly, points get removed from the possible set of endpoints when their Kriging variance falls below the mean of the variance field. This method considers the mission time and minimum expectation of the

measurement quality by re-planning and using a Traveling Salesperson (TSP) algorithm. In this thesis, minimum expected measurement quality is set before an exploration by tuning the maximum allowed area to scan.

## 1.2 Problem Definitions

The problem space will be defined in an effort to be consistent in naming conventions and parameter definitions throughout this work. The conventions described in Section 1.2 will be used throughout the rest of the work.

### 1.2.1 Notation

A boldface lowercase letter, for example,  $\mathbf{v}$ , will denote a column vector of real numbers. An non-boldface uppercase letter, for example,  $M$ , will denote a two-dimensional matrix of real numbers.

### 1.2.2 The Field

The initially unknown field, referred to as the *target field*, is a rectangular field of height  $h$ , and width  $w$ , i.e.  $Z \in \mathbb{R}^{h \times w}$ . The field is made up of square pixel cells, referred to as *vesicles*. Each vesicle can be “visited”, or sampled, in order to yield a single state of interest in the set of real numbers. Throughout this thesis a square target field (i.e.  $h = w$ ) will be used, and  $h$  and  $w$  will be natural numbers.

### 1.2.3 The Sensor

The observations of interest made on the field will be using ideal sensors with no measurement noise. The sensors will measure a subset of the area of the entire target field. This area will be referred to as the *sensor footprint*, and will be equal to the size of a single vesicle of the target field.

The locations of the sensor measurements must be known for the methods developed. The locations of the measurements will be represented as Cartesian coordinates on the field. For an arbitrary observation of the field, the location of the measurement will be at corresponding coordinates  $\mathbf{s} \in \mathbb{R}^2$ , and the sensor measurement would be  $Z(\mathbf{s})$ . The value of  $Z$  at  $\mathbf{s}$  is quantized to the vesicle in which the point  $\mathbf{s}$  falls within.

### Real World Sensing Examples

A Global Positioning System (GPS) sensor would likely be used to estimate localized position of a sensor measurement on Earth. In the case of predicting the boundaries of a glacier, for example, an infrared sensor would likely be used to measure the state of interest, thermal output of the field in this case. In the case of terrain mapping a LiDAR sensor could be used to sample terrain altitudes of the terrain below, at marked locations using GPS, on a UAV.

## Chapter 2

# Spatial Analysis

Many of the methods introduced in this thesis will rely on works developed in the fields of Spatial Statistics, Geostatistics, and Geography. Geo-statisticians have developed much of the work surrounding field predictions in the geospatial domain. The Kriging Method, a best linear unbiased predictor (BLUP) produces a prediction based on statistical data gathered from samples taken on a field. The Kriging Method predicts the state of a point on a field from weights generated from a covariance matrix created from samples on the field. A variance for each computed prediction can also be generated as a byproduct of the Kriging prediction, and this will be used to calculate information gain in the path finders introduced in this paper. It is assumed that the expected value of each point is from a normal distribution, where the variance and expected value of the distribution is a function of the neighboring samples and spatial autocorrelation factor of the field.

Tobler's First Law of Geography [24] states, "Everything is related to everything else, but near things are more related than distant things." Regarding geospatial data, there is

a positive correlation between observations with a small difference in distance [14]. This implies the existence of geospatial autocorrelation in many target fields of interest, where there exists a positive correlation between elements in a field. Geospatial autocorrelation is the hypothesis that allows naive prediction techniques, like Inverse Distance Weighting (IDW) (Section 2.1), to work. The Kriging Method first finds the underlying spatial autocorrelation of a target field from a set of samples, and then predicts the state of a given vesicle by emphasizing values of statistically similar samples in a weighted sum. The methods introduced in this chapter are intended to serve as an introduction and background into the Kriging Method.

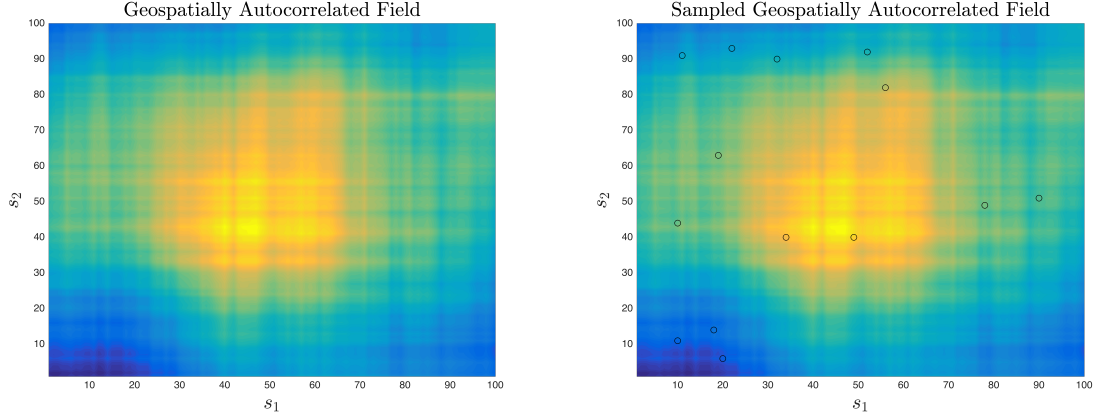
### **2.0.1 Autocorrelation in a Field**

Positively correlated spatial autocorrelation in a field implies the existence of a cluster of similar points near one another i.e. relatively small covariances between two spatially similar points. The opposite is true when the overall spatial autocorrelation of a field is negative. Using Tobler's First Law of Geography, along with the assumption that fields contain positive autocorrelation distributed uniformly throughout a field, the degree of spatial autocorrelation in a field can be measured, and will be discussed in Section 2.2 on Variography.

## **2.1 Inverse Distance Weighting**

An inverse distance weighting is a naive interpolation tool where a point is predicted based its distances from a set of observed points. A simple IDW, using Shepard's Method





(a) A randomly generated spatially autocorrelated field. (b) Samples at marked locations were taken of the target field in 2.1a.

Figure 2.1: A Gaussian distributed randomly generated spatially autocorrelated field.

[22], gives a prediction,  $\hat{Z}(\mathbf{s}_j)$ , of an unobserved point,  $\mathbf{s}_j$ , as a function of the  $N \in \mathbb{N}$  observed points,  $\{Z(\mathbf{s}_1), Z(\mathbf{s}_2), \dots, Z(\mathbf{s}_n)\}$ .

$$\hat{Z}(\mathbf{s}_j) = \begin{cases} \frac{\sum_{i=1}^N [w(\mathbf{s}_j, \mathbf{s}_i)] Z(\mathbf{s}_i)}{\sum_{i=1}^N w(\mathbf{s}_j, \mathbf{s}_i)} & \text{if } \forall i \mid d(\mathbf{s}_j, \mathbf{s}_i) \neq 0 \\ Z(\mathbf{s}_j) & \text{if } \exists i \mid d(\mathbf{s}_j, \mathbf{s}_i) = 0 \end{cases} \quad (2.1)$$

$$d(\mathbf{s}_j, \mathbf{s}_i) = \|\mathbf{s}_j - \mathbf{s}_i\|_2 \quad (2.2)$$

$$w(\mathbf{s}_j, \mathbf{s}_i) = \frac{1}{d(\mathbf{s}_j, \mathbf{s}_i)^p} = \|\mathbf{s}_j - \mathbf{s}_i\|_2^{-p} \quad (2.3)$$

where  $p \in \mathbb{R}_{\geq 0}$  is the IDW “power parameter”. The power parameter,  $p$ , controls the emphasis on near and far observations on a prediction. As  $p$  increases, the predicted values more closely resemble the closest made observation to the prediction location. Inversely, as  $p$  gets smaller within  $(0, 1]$ , more emphasis is drawn from observations made further away.

This method can yield a prediction for all possible points in a field where a set of observations at known locations are made, as done in Figure 2.2. Unfortunately, the method

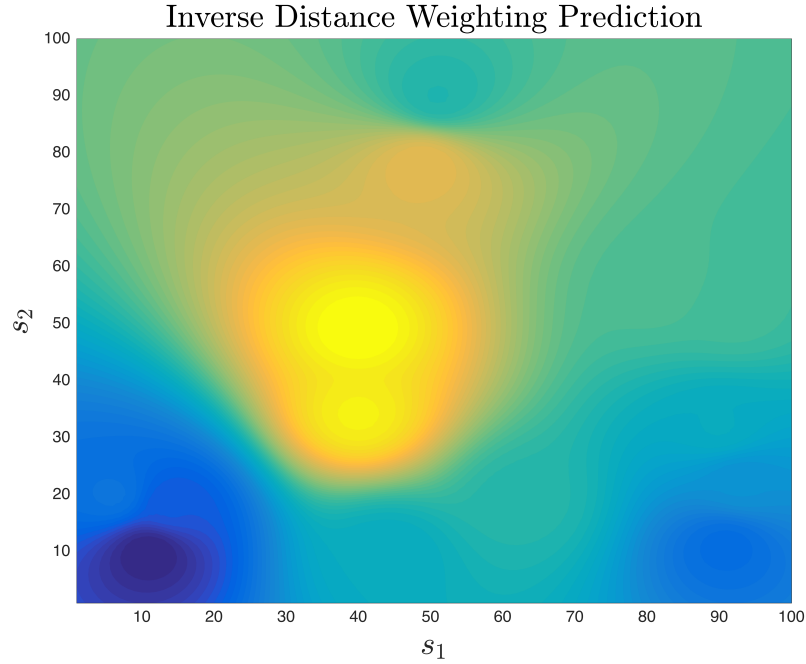


Figure 2.2: An inverse distance weighting predicted field generated from the samples taken of Figure 2.1a at the locations marked in Figure 2.1b.

is limited in that it assumes a spherical distribution of correlation of points in a field, and does not take advantage of the underlying spatial correlation patterns of the target field being observed to make a more methodical weighted sum prediction. A field that exhibits properties of spatial autocorrelation would be more statistically exploitable because the distribution of the states of interest on the field can be learned.

## 2.2 Variography

Variography is a set of procedures for examining and interpreting spatial dependence and spatial autocorrelation in a field of observed data. The *variogram* of a field will be introduced to assist in extracting the underlying spatial autocorrelation function of a target

field. The variogram function will be factored into a classical prediction via weighting, yielding a Kriging Weighting.

### 2.2.1 The Variogram

A variogram quantifies dependence for two disjoint observations separated by some distance, or *lag*, away. The function, in essence, yields a value directly proportional to the covariance between two given points in a stochastic field. The autocorrelation is assumed to be independent of direction so there is a single variogram for all points in the field.

A Variogram is intended to be a continuous function which yields a covariance between two points  $Z(\mathbf{s}_i)$ ,  $Z(\mathbf{s}_j)$ , which have not necessarily been observed, but known to be a Euclidean distance, or lag,  $h_{i,j} \in \mathbb{R}$  apart [4], where

$$h_{i,j} = \|\mathbf{s}_i - \mathbf{s}_j\|_2 \quad (2.4)$$

Following Equation 2.4.1 of Matheron, 1963 [13], the value of a point on a field is

$$Z(\mathbf{s}_i) = \mu(\mathbf{s}_i) + \theta(\mathbf{s}_i) \quad (2.5)$$

where  $\theta(\cdot)$  is a zero-mean intrinsically stationary stochastic Wiener process, and  $\mu(\cdot) = \bar{Z}$  is the mean value of the state of interest in the field.

### 2.2.2 The Semivariogram

The Semivariogram is defined to be the average squared difference between two points separated by some distance apart. Matheron, 1963 defines a semivariogram in [13] in three-dimensional space, which reduces to the following for two dimensions

$$\gamma(h) = \frac{1}{2A} \iint_A [Z(\mathbf{s} + h) - Z(\mathbf{s})]^2 dA \quad (2.6)$$

where  $A$  is a closed area in a field to consider,  $Z(\mathbf{s})$  is the value of a point at location  $\mathbf{s}$  on the field, and  $Z(\mathbf{s} + h)$  is the value of some point a distance  $h$ , defined in Equation 2.4, apart from a point  $\mathbf{s}$  on the field.

It is infeasible to estimate an observation value at each possible point in the field to compute a continuous Semivariogram. Furthermore, the fields observed using these methods are typically gridded, and therefore not continuous by their analytical nature. A discrete model must first be constructed, and will then be fit into a continuous variogram model. This is done by first constructing a discrete variogram model, or *Empirical Semivariogram*, and then fitting a continuous model to it. Fitting a discrete Semivariogram should in turn yield a function close to  $\gamma(h)$  defined in Equation 2.6, and should be identical assuming every point in the area  $A$  is sampled with infinite precision.

### 2.2.3 The Empirical Semivariogram

An Empirical Semivariogram, or Experimental Semivariogram, is a discrete function representing the covariance of the observation value difference between two sampled locations that are some distance  $h$  apart. Goovaerts defines the empirical variogram in *Geostatistics for Natural Resources Evaluation. Applied Geostatistics Series* [23] as:

$$\hat{\gamma}(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} (Z(\mathbf{s}_i) - Z(\mathbf{s}_i + h))^2 \quad (2.7)$$

where  $N(h)$  is the cardinality of the set of all pairs of observed points that are a Euclidean distance, or lag,  $h$ , apart.

The experimental variogram conveys the spatial autocorrelation of a sampled field. As the lag between two given points increases, the covariance also increases when the field

is spatially autocorrelated. The covariance levels out to a steady value (the *sill*) at some distance in the domain (the *range*). The range marks the point where the loss of reliable spatial autocorrelation between two points ceases.

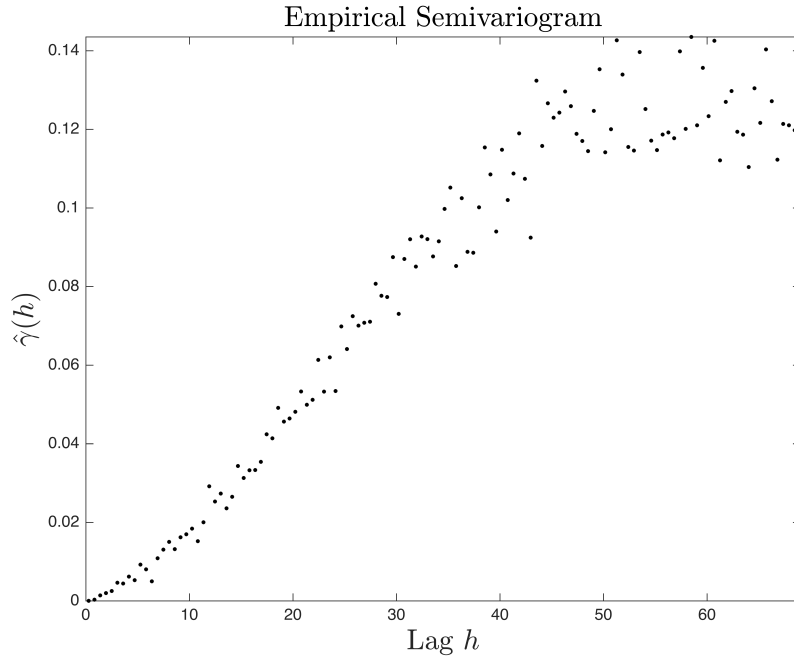


Figure 2.3: An empirical semivariogram.

#### 2.2.4 Converting a Semivariogram to a Variogram

The intent of fitting a statistical model to an experimental variogram is to approximate the continuous covariance for any two points, that have not necessarily been observed, on  $Z$  that are at some known lag apart.

The Empirical Semivariogram will be fit to a statistical model, or *kernel*, known as a Variogram Model. There exist well-known models, further discussed in this section. Each model is a scalar function of lag,  $h$ , sill,  $s$ , and range,  $a$ . The term *sill* refers to the point

on the co-domain where two points at the lag specified are no longer autocorrelated. The sill is therefore the largest value of covariance for two disjoint points on a field that are still considered to be autocorrelated. The corresponding point on the domain for the sill is referred to as the *range* on the variogram. Two points that have a lag larger than the range are not considered to be autocorrelated. The *nugget* of the variogram is defined to be the variance at zero lag, or  $\gamma(0)$  [13]. This value is exactly zero for ideal measurements. For non-ideal situations, the nugget is typically non-zero. This can be attributed to drift in the the field states between sampling periods, or from measurement noise. The value found for the nugget is summed with the value yielded by  $\gamma$ , to get the final variogram value for a given lag [23].

Three kernel models used in this thesis are the Gaussian, Exponential, and Spherical models.

### **The Gaussian Model**

$$\gamma_g(h, s, a) = s \left[ 1 - \exp \left( - \frac{h^2}{a^2} \right) \right] \quad (2.8)$$

The Gaussian model will asymptotically reach its sill. The sill would be at the limit as  $h$  approaches infinity. The *practical range* is therefore used to refer the point on the domain where the variogram reaches 95% of its sill [23].

### **The Exponential Model**

$$\gamma_e(h, s, a) = s \left[ 1 - \exp \left( - \frac{h}{a} \right) \right] \quad (2.9)$$

The same rules as the Gaussian model apply to the Exponential model [23].

## The Spherical Model

$$\gamma_s(h, s, a) = \frac{s}{2} \left[ \frac{3h}{a} - \left( \frac{h}{a} \right)^3 \right] \quad (2.10)$$

The spherical model will reach an exactly zero slope at the sill and range [23].

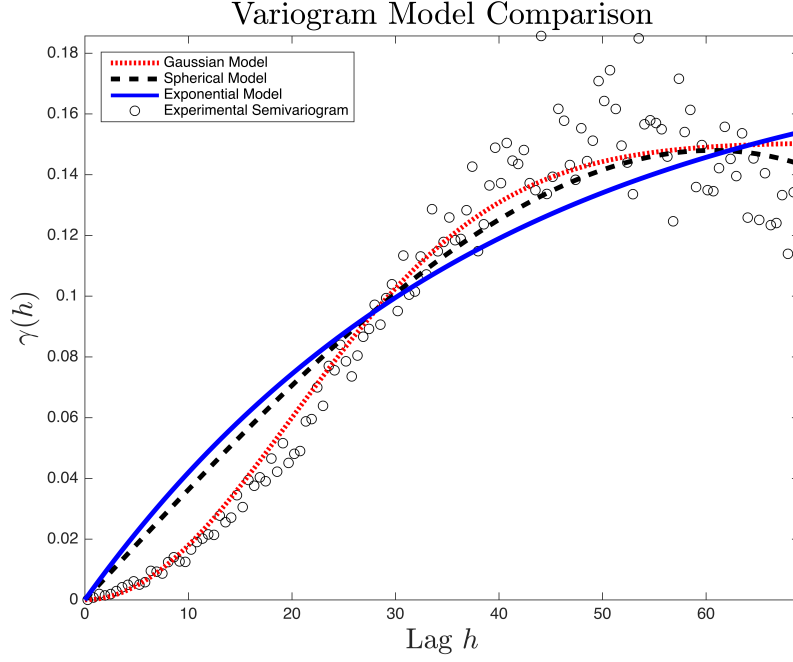


Figure 2.4: Examples of three different variogram models.

### 2.2.5 Fitting a Semi-Variogram

The kernel function of the range,  $a$ , the sill,  $s$ , and lag,  $h$  is chosen based on the statistical properties of the field being examined. Although there exist no closed form solution for finding an appropriate variogram model for a given field, one can compare a variety of different models against one another. Conducting cross-validation tests and comparing root-mean squared prediction errors for different models are common approaches for finding appropriate variogram models.

Using a version of the *fminsearch* function in *MATLAB*, a variogram can be fit to the desired objective function from a set of samples and initial guesses for the range and sill using a simplex search method. As the function is used over several iterations of sampling, the fit range and sill values found in the previous iteration can be used as the seed to the next iteration of the fit in an attempt to minimize computation time. The *MATLAB* function, *fminsearch*, is defined to “find the minimum of an unconstrained multi-variable function using a derivative-free method” [9], expressed in Equation 2.11.

$$\gamma(h) = \min [\gamma_{kernel}(h, s, a) - \hat{\gamma}(h)]^2 \quad (2.11)$$

The function is then modified by specifying bounds of minimization in an attempt to decrease iterations of the function fit, which can be computationally expensive as more samples are taken. This modified version of *fminsearch*, named *fminsearchcon*, can be downloaded from the MathWorks File Exchange.

## 2.3 The Kriging Method

The Kriging Method conducts a weighted sum using the continuous variogram model that was fit to the physical observations made. The method can yield a prediction for each vesicle in a target space similar to the Inverse Distance Weighting method described in Section 2.1, but with more statistical robustness.

### 2.3.1 Forms of the Kriging Method

There are three major forms of the Kriging Method; all of which differ primarily in the handling of the mean gathered from observations of a target field. The *Simple Kriging*



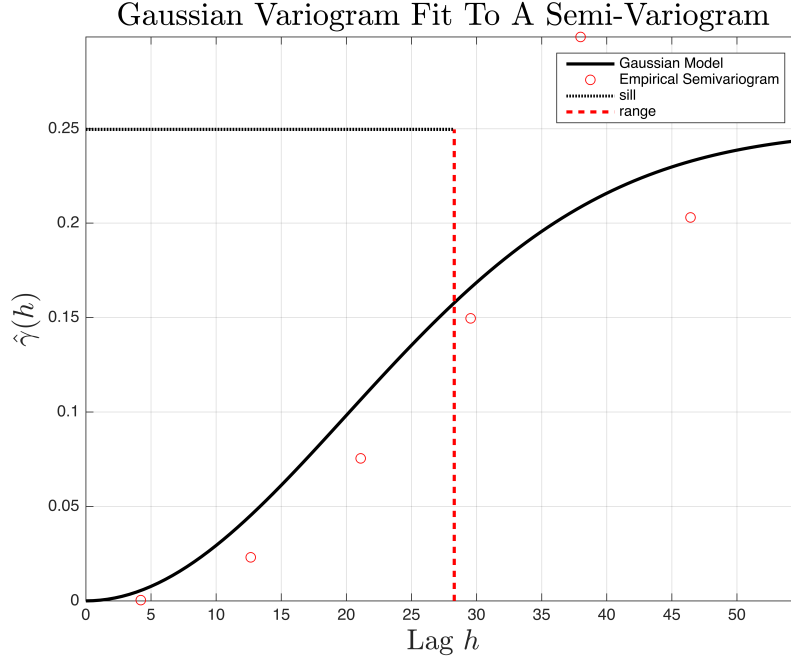


Figure 2.5: An experimental variogram generated using Equation 2.7 from the samples taken in Figure 2.1b.  $\delta$  was chosen such that for  $n$  observations, a total number of  $\left\lfloor \frac{n}{2} \right\rfloor$  points were plotted. A Gaussian statistical model was fit to the experimental variogram. The variogram was fit using *fminsearchcon* in *MATLAB*.

*Method* makes the assumption that the mean is known and constant throughout the entirety of an observed field. This is not the case for fields that are very large as it does not follow Tobler’s First Law. The *Ordinary Kriging Method* can deduce the local mean of a neighborhood from a smaller subset of observations in a larger target field. This is done by classifying the larger field into smaller neighborhoods where the mean is only constant within those neighborhoods. Ordinary Kriging has the advantage that the mean is not required to be known before running a prediction. The *Universal Kriging Method* can perform similar local mean calculations as the Ordinary Kriging Method, but does so by fitting a polynomial representing a mean trend model and not from a constant mean value representing that neighborhood [25] as seen in Section 2.2.5 on fitting a variogram. The

Ordinary Kriging method will be used throughout the rest of this thesis because of its lack of requirement for the expected value of the field, and its computational simplicity when compared to the Universal Kriging Method.

### 2.3.2 Covariance Matrix from a Variogram

From the fit variogram which represents the spatial statistics of a field from a set of samples, a variance-covariance matrix for  $N$  observations,  $P \in \mathbb{R}^{N \times N}$ , will be constructed. The value of the element  $P_{i,j}$ , will represent the covariance of the lag between the  $i^{th}$  and  $j^{th}$  observations on the field [23], [13]. If  $i = j$ , the value of the element,  $P_{i,j}$  is the variance of the  $i^{th}$  observation.

$$P_{i,j} = \text{cov}\{Z(\mathbf{s}_i), Z(\mathbf{s}_j)\} = \gamma(\|\mathbf{s}_i - \mathbf{s}_j\|_2) \quad (2.12)$$

$$P = \begin{bmatrix} \gamma(0) & \gamma(\|\mathbf{s}_1 - \mathbf{s}_2\|_2) & \dots & \gamma(\|\mathbf{s}_1 - \mathbf{s}_N\|_2) \\ \gamma(\|\mathbf{s}_2 - \mathbf{s}_1\|_2) & \gamma(0) & \dots & \gamma(\|\mathbf{s}_2 - \mathbf{s}_N\|_2) \\ \vdots & \vdots & \ddots & \vdots \\ \gamma(\|\mathbf{s}_N - \mathbf{s}_1\|_2) & \gamma(\|\mathbf{s}_N - \mathbf{s}_2\|_2) & \dots & \gamma(0) \end{bmatrix} \quad (2.13)$$

### Numerical Precision Consideration

As the number of observations,  $N$ , made on a field become large, the matrix,  $P$ , grows on the order of  $N^2$ . For a 30% scan of a size  $100 \times 100$  field,  $P$  grows to be a matrix of size  $3000 \times 3000$ . A matrix with a size on this order of magnitude is likely to be ill-conditioned, i.e. the matrix contains columns that are close to the linear combinations of the other columns in the matrix. A high condition number implies that  $PP^{-1}$  deviates from the identity matrix of the same size. The inverse of such a matrix, used in Equation 2.15, will be prone to numerical imprecisions using *MATLAB*'s standard inverse function, which uses a lower-

upper (LU) decomposition [10]. When taking the inverse of such a matrix for the purposes of this thesis, the Moore-Penrose pseudo-inverse of  $P$ ,  $P^\dagger$ , is taken, in *MATLAB*. The pseudo-inverse function in *MATLAB* computes the Singular Value Decomposition (SVD) of the matrix [11], which is more computationally expensive, but more robust to numerical errors when inverting an ill-conditioned matrix. All computational matrix inversions in this thesis will be calculated using a pseudo-inverse via an SVD.

### 2.3.3 Mean & Variance of a Point Prediction

For any given point on a field, we can construct a *proximity vector*,  $\mathbf{d}_0 \in \mathbb{R}^N$ , which contains the covariance of a given point,  $\mathbf{s}_0$  on the field with the  $N$  observations made. The  $k^{th}$  element of  $\mathbf{d}_N$ , would therefore contain the covariance for the lag between point  $\mathbf{s}_0$  and the  $k^{th}$  observation made,  $\mathbf{s}_k$  [13].

$$\mathbf{d}_0(k) = \gamma(\|\mathbf{s}_0 - \mathbf{s}_k\|_2)$$

$$\mathbf{d}_0 = \begin{bmatrix} \gamma(\|\mathbf{s}_0 - \mathbf{s}_1\|_2) \\ \gamma(\|\mathbf{s}_0 - \mathbf{s}_2\|_2) \\ \vdots \\ \gamma(\|\mathbf{s}_0 - \mathbf{s}_N\|_2) \end{bmatrix} \quad (2.14)$$

Furthermore, the Kriging Method can be bounded. If a to-be-predicted point and a given sample is beyond the range value fit to the variogram model, the corresponding element in the proximity vector is set to the sill. This ensures that points outside of the range of autocorrelation are not weighted anymore than they should be. This method is suggested when the variogram model used is a bounded function, e.g. the Spherical Model (Equation 2.10) which is the model used in this thesis.

A set a weights will be computed for each vesicle in the target field similarly to the Inverse Distance Weighting method. These weights will be referred to as the *Ordinary Kriging Weights*. For a given prediction location,  $\mathbf{s}_0$ , the Ordinary Kriging Weight vector,  $\boldsymbol{\lambda}_0$ , will be defined as the product of the inverse of the covariance matrix of the field and the proximity vector of the point to predict [6].

$$\begin{bmatrix} \boldsymbol{\lambda}_0 \\ \eta_0 \end{bmatrix} = \begin{bmatrix} P^{-1} & \mathbf{1} \\ \mathbf{1}^T & 0 \end{bmatrix} \begin{bmatrix} \mathbf{d}_0 \\ 1 \end{bmatrix} \quad (2.15)$$

where  $\eta_0$  is a Lagrangian multiplier and  $\mathbf{1} \in \mathbb{R}^N$  is a vector of 1s. These terms assist the Ordinary Kriging system in maintaining unbiasedness in predictions by forcing the sum of the Kriging Weights,  $\lambda_0$ , to one [6].

The Ordinary Kriging equation will be used to predict the value,  $\hat{Z}(\mathbf{s}_0)$  of an unobserved location,  $\mathbf{s}_0$ . The prediction is a function of the Kriging Weights and a vector of  $N$  observations [6].

$$\hat{Z}(\mathbf{s}_0) = \begin{bmatrix} Z(\mathbf{s}_1) & Z(\mathbf{s}_2) & \dots & Z(\mathbf{s}_N) \end{bmatrix} \boldsymbol{\lambda}_0 \quad (2.16)$$

The variance of a point predicted on a target field can be calculated using byproduct terms generated along the way of calculating a Kriging prediction [6]. For a predicted point  $\hat{Z}(\mathbf{s}_0)$ , using the proximity vector,  $\mathbf{d}_0$ , defined in Equation 2.14, and the Kriging Weights,  $\boldsymbol{\lambda}_0$  defined in Equation 2.15 for the predicted point, the variance of the prediction for that point is defined as:

$$\text{var}\{\hat{Z}(\mathbf{s}_0)\} = \begin{bmatrix} \mathbf{d}_0 \\ 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{\lambda}_0^T & \eta_0 \end{bmatrix} \quad (2.17)$$

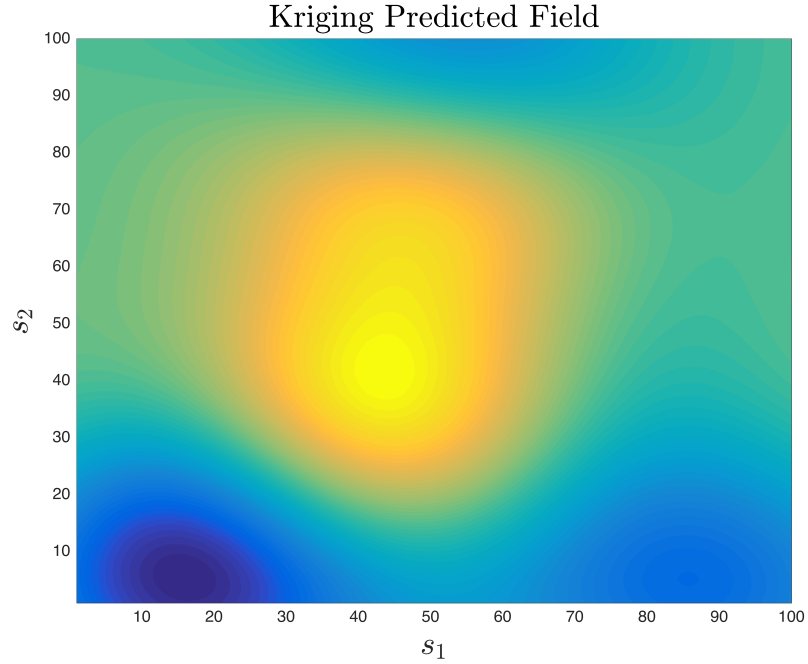


Figure 2.6: A Kriging Method predicted field generated from the samples taken of Figure 2.1a at the locations marked in Figure 2.1b.

#### 2.3.4 Procedure for Field Prediction Using the Kriging Method

The Kriging Prediction is run at every possible unobserved vesicle in the target field in order to predict the entirety of a target field from a finite set of  $N$  observations and their respective locations,  $O$ .

When Algorithm 1 is run on the target field from Figure 2.1a, for the samples taken in Figure 2.1b, a prediction of the entire field can be generated, as seen in Figure 2.6.

---

**Algorithm 1:** Kriging Prediction of Target Field

---

```

1: procedure KRIGINGPREDICTFIELD( $Z, O$ )
2:   Generate Semi-Variogram:
3:      $\forall \mathbf{s}_i, Z(\mathbf{s}_i) \in O$ :
4:        $\hat{\gamma}(h) \leftarrow \mathbf{s}_i, Z(\mathbf{s}_i)$ 
5:
6:   Generate Variogram:
7:      $\gamma(h)$  fits to  $\hat{\gamma}(h)$ 
8:
9:   Construct Covariance Matrix:
10:     $\forall (\mathbf{s}_i, \mathbf{s}_j) \in O$  :
11:       $h_{i,j} = \|\mathbf{s}_i - \mathbf{s}_j\|_2$ 
12:       $P_{i,j} = \gamma(h_{i,j})$ 
13:
14:     $\forall i \in [1, N]$  :
15:       $P_{i,N+1} = 1$ 
16:       $P_{N+1,i} = 1$ 
17:       $P_{N+1,N+1} = 0$ 
18:
19:   Run Kriging Predictions For Target Field:
20:     $\forall \mathbf{p}_i \in \text{field}$ :
21:       $\mathbf{d}_i = \begin{bmatrix} \gamma(\|\mathbf{s}_1 - \mathbf{p}_i\|_2) \dots \gamma(\|\mathbf{s}_N - \mathbf{p}_i\|_2) & 1 \end{bmatrix}^T$ 
22:       $\begin{bmatrix} \boldsymbol{\lambda}_{\mathbf{p}_i} \\ \eta_{\mathbf{p}_i} \end{bmatrix} = \begin{bmatrix} P^{-1} & \mathbf{1} \\ \mathbf{1}^T & 0 \end{bmatrix} \begin{bmatrix} \mathbf{d}_{\mathbf{p}_i} \\ 1 \end{bmatrix}$ 
23:       $\hat{Z}(\mathbf{p}_i) = \begin{bmatrix} Z(\mathbf{s}_1) \dots Z(\mathbf{s}_N) \end{bmatrix} \boldsymbol{\lambda}_{\mathbf{p}_i}$ 
24:       $\text{var}\{\hat{Z}(\mathbf{p}_i)\} = \begin{bmatrix} \mathbf{d}_i \\ 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{\lambda}_{\mathbf{p}_i}^T & \eta_{\mathbf{p}_i} \end{bmatrix}$ 

```

---

## Chapter 3

# Path Planning

The goal of each of the planners introduced is to assist in the discovery of a field's features with an adjustable trade off between speed and confidence of prediction. The user of such a system could choose to scan more area if fuel is not of high concern. Likewise, if the field is very large, or several fields need to be scanned in a limited amount of time, a quicker scan with a lower degree of prediction certainty can be performed.

### 3.1 Field Uncertainty Model

The hypothesis that intentionally suppressing prediction variance yields a higher quality field prediction is used as the basis of the path planners introduced. The root mean square (RMS) error of any estimator is composed of two parts: a bias and variance of the estimate about the bias. The RMS error is reduced by reducing the variance of estimates. A Best Linear Unbiased Prediction method, such as the Kriging method, can therefore produce higher quality estimates with lower prediction variances.

A method for calculating the variance of a prediction was defined as a function of the proximity vector and Kriging weights generated for the prediction point in Equation 2.17. For points that have been directly measured, the variance is ideally zero (for fields with no drift or dynamics). The uncertainty of the prediction of a point in the target field is the variance of its prediction. The goal of a path planner intending to suppress uncertainty of all predictions in a target field would be to reduce the overall variance of the target field being explored.

Let  $\Sigma(\cdot)$  be a criterion for overall predicted field uncertainty. The function can be defined as the average variance calculated from a prediction of all  $h \times w$  predictable points on a target field from a set of observations,  $S$ .

$$\Sigma(\hat{Z}_S) = \frac{1}{hw} \sum_{i=1}^{hw} \text{var}\{\hat{Z}_S(\mathbf{p}_i)\} \quad (3.1)$$

where  $\Sigma(\hat{Z}_S) \in \mathbb{R}_{\geq 0}$  and  $\text{var}\{\hat{Z}_S(\mathbf{p}_i)\} \in \mathbb{R}_{\geq 0}$  is the variance of the prediction of the  $i^{th}$  point,  $\mathbf{p}_i$ , when the field is predicted from a set of samples,  $S$ .

### 3.1.1 Uncertainty Loss Function

A criterion for overall field uncertainty was introduced in Section 3.1. Given a set of sampled points,  $S$  on a field, the overall field uncertainty is the mean variance of all points on the field,  $\Sigma(\hat{Z}_S)$ . For an additional set of samples,  $T$ , taken on the field, a new field uncertainty,  $\Sigma(\hat{Z}_{S \cup T})$ , is the field uncertainty criterion of the fields prediction from the union of the sample sets  $S$  and  $T$ . The difference in overall field uncertainty,  $L(T)$ , will be defined as the uncertainty lost by taking the additional samples in the set  $T$  on the field.

$$L(T) = \Sigma(\hat{Z}_S) - \Sigma(\hat{Z}_{S \cup T}) \quad (3.2)$$



## 3.2 Path Planning Overview

Five variance suppressing path planners are introduced in this thesis. Each of five path planners attempts to reduce Kriging prediction variance by steering an exploration vehicle through a target field in a fashion that is predicted to reduce overall field uncertainty. All five of the path planners introduced will need an initial set of samples to make an initial path decision. Each of the five path planners begin by conducting an initial sweep on the main diagonal of the field. They initially stop at a waypoint set to a point close to the middle point on the field. The first point is the point in which the zig-zag method (discussed in Section 3.8.1) initially stops. The initial set of samples taken from the sweep will then be used to make an initial decision.

## 3.3 Highest Variance Path Planner

The *Highest Variance* (HV) Path Planner attempts to reduce field prediction uncertainty by setting the exploration vehicle's destination to the point of highest prediction variance on the field. After meeting the point of highest variance, the field prediction and variances are recalculated from the samples the vehicle took on its path to the previously selected destination point. The next destination, or *decision point*, is then set to the new point of highest prediction uncertainty.

Sampling the location of the highest variance is the simplest and most naive approach to path planning using the Kriging method. The highest point of uncertainty on the field is the point,  $\mathbf{p}$ , is defined as:

$$\arg \max_{\mathbf{p}} \text{var}\{\hat{Z}(\mathbf{p})\} \quad (3.3)$$

By simply setting the next decision point of the path to  $\mathbf{p}$ , the point of highest uncertainty will be sampled at the end of the path. Once the point is met at the end of the path, a new set of samples gathered from the path to the endpoint will be used to recalculate the statistical patterns of the field to higher degree of quality. A Kriging prediction, variances of those predictions are then run on the field. The path planner continues by setting the next decision point to the point of highest uncertainty after recalculating the variances of the field. The planner terminates exploration once a preset maximum scan area limit has been met by the exploration vehicle.

### 3.3.1 Inefficiency in Highest Variance Method

The HV algorithm does not account for repeating paths, or avoiding the re-sampling of points on the field. The only knowledge used is the variance of the endpoint of a path. Although the ground covered by the algorithm may be sufficient for uncertainty suppression, a path planner that considers the cost of trajectories would likely yield better results.

## 3.4 $N$ Highest Variances Path Planner

The  $N$  *Highest Variances* (N-HV) Path Planner sets its decision point to a point from a set of the  $N$  points of highest prediction variances. A *leg*, or trajectory between the current position of the exploration vehicle and a potential decision point is calculated for all points in the set. The leg that is predicted to reduce the most overall field uncertainty (Equation 3.1) is set as the next decision point of the vehicle. When the decision point is met, the

set of legs to the  $N$  highest variances is recalculated. The next decision point is set to the point that yields the leg that is expected to maximize loss in field uncertainty.

Let  $K_N$  be the set of the  $N \in \mathbb{N}$  points of highest uncertainty on the field. Let  $T_i$  be a candidate trajectory connecting the current position of the exploration vehicle to the  $i^{th}$  point in the set  $K_N$ . The endpoint that is ultimately chosen by the  $N$  Highest Variance Path Planner is the one that maximizes the loss function,  $L(T_i)$ . The points along a trajectory,  $T_i$ , have likely not been sampled, as they represent points of high uncertainty. The loss in uncertainty for taking the path,  $T_i$ , is therefore not known. An estimate of the loss in overall field uncertainty,  $\hat{L}(T_i)$ , after taking the path,  $T_i$ , is calculated by using the previous Kriging predictions of the points along the path. The predictions of those points are used as actual samples taken on the field in a new Kriging prediction variance calculation of the field.

### 3.5 Monte Carlo Path Planner

The Monte Carlo Path Planner (MCP) calculates a set of legs to the  $N$  highest points of variance on the field, similarly to the  $N$ -HV planner, except that for each leg, a separate set of  $M_{mc}$  random trajectories are calculated around the leg. The exploration vehicle trajectory, or set of waypoints to a decision point, that is selected, is the random trajectory that maximizes loss in field uncertainty. The samples taken along the last trajectory selected are stored and used to recompute field variances when the final point in the trajectory (decision point) is met. New possible trajectories are then recalculated, and the planner repeats until a predefined maximum exploration distance has been met by the vehicle. This

method compares a total of  $NM_{mc}$  noisy trajectories per decision point met. By introducing noise into each of the trajectories found in the  $N$ -HV path planner, a more optimal path may be found.

Let  $K_N$  be the set of the  $N$  points of highest prediction variances on the field. Let  $T_i$  be a candidate trajectory from the current position of the exploration vehicle to the  $i^{th}$  endpoint in the candidate endpoint set,  $K_N$ . Each point in the candidate trajectory is a waypoint the exploration vehicle will visit on its way to the last point in the sequence. The trajectory is a set of states representing the field position,  $(x, y)$ , and the vehicle heading angle,  $theta$ . The  $k^{th}$  vector in the state candidate trajectory set,  $T_i$ , denoted as  $T_i(k)$ , will be the state the exploration vehicle will take on at that position on the field, i.e.

$$T_i(k) = \begin{bmatrix} x_i(k) \\ y_i(k) \\ \theta_i(k) \end{bmatrix} \quad (3.4)$$

Let  $\alpha \in \mathbb{R}$  be the step size of the vehicle from one point to the next within the trajectory,  $T_i$ . Let  $\mathbf{w}_i \in \mathbb{R}^2$  be a vector of two zero-mean Wiener processes with a tunable process standard deviation which is less than the step size,  $\alpha$ . Furthermore, the step size of the vehicle,  $\alpha$ , can not be a value less than the distance the exploration vehicle can travel in one time-step.

$$\text{var}\{\mathbf{w}_i\} = \begin{bmatrix} \text{var}\{w_{i_x}\} \\ \text{var}\{w_{i_y}\} \end{bmatrix} \quad (3.5)$$

$$\text{var}\{w_{i_x}\} < \alpha^2 \quad (3.6)$$

$$\text{var}\{w_{i_y}\} < \alpha^2 \quad (3.7)$$

The corresponding Monte Carlo path, or sequence of waypoints the exploration vehicle will make on its way to the candidate endpoint,  $\mathbf{p}_i = [p_x \ p_y]^T$ .

$$T_i(k) = \begin{cases} \begin{bmatrix} x_0 \\ y_0 \\ \text{atan2}(p_y - y_0, p_x - x_0) \end{bmatrix} & : k = 1 \\ \begin{bmatrix} \alpha \cos \theta_i(k) \\ \alpha \sin \theta_i(k) \\ \text{atan2}(p_y - y_i(k), p_x - x_i(k)) \end{bmatrix} + \begin{bmatrix} \mathbf{w}_i(k) \\ 0 \end{bmatrix} & : 1 < k < \left\lceil \frac{\|\mathbf{p} - \mathbf{s}\|_2}{\alpha} \right\rceil \\ \begin{bmatrix} p_x \\ p_y \\ 0 \end{bmatrix} & : k = \left\lceil \frac{\|\mathbf{p} - \mathbf{s}\|_2}{\alpha} \right\rceil \end{cases} \quad (3.8)$$

where the initial point in the candidate trajectory is set to the current position,  $\mathbf{s} = [x_0 \ y_0]^T$  from the state vector of the exploration vehicle. Due to the uncertainty in length of the random trajectory generated, the number of waypoints in the candidate trajectory  $T_i$  is fixed, such that  $k \in [1, \left\lceil \frac{\|\mathbf{p} - \mathbf{s}\|_2}{\alpha} \right\rceil]$  (the number of points in the trajectory for a step size,  $\alpha$ , given zero variance noise added to the process). The last point in the candidate trajectory set is set to the corresponding endpoint from the set  $K_N$ .

As introduced in the  $N$ -HV path planner, a candidate trajectory is generated for each of the candidate endpoints in the set,  $K_N$ . The candidate trajectory that maximizes the loss function,  $L(T)$ , is the path that is ultimately selected as the next vehicle trajectory. In an effort to find a more optimal trajectory, more than one random walk can be generated for each endpoint in the set  $K_N$ . The variable  $M_{mc} \in \mathbb{N}$  will denote the number of random walks taken per endpoint in the set  $K_N$ .

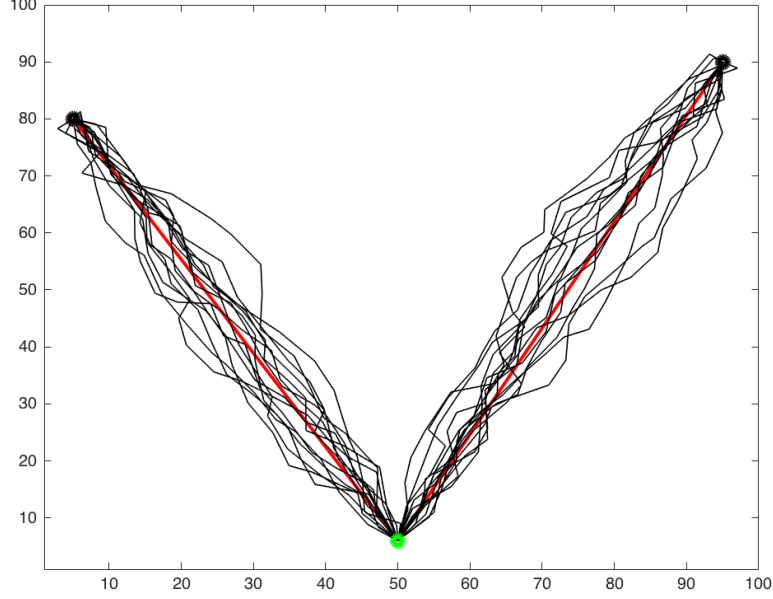


Figure 3.1: Monte Carlo paths (black) surrounding deterministic  $N$ -HV paths (red). The starting point,  $(\mathbf{s} = [50 \ 6]^T)$ , is indicated in green. The set  $K_N$  contains  $N = 2$  endpoints ( $K_N = [5 \ 80]^T, [95 \ 90]^T$ ).  $M_{mc} = 15$  random walks are generated for each endpoint.  $\alpha = 5$ . The variance of the Wiener process states are  $\frac{1}{2}\alpha$ .

### 3.6 The Gradient Ascent Path Planner

The Gradient Ascent (GA) Path Planner reduces field prediction variance by maximizing the movement in the local variance field's gradient at every move. At every decision point, a circle, with radius  $r \in \mathbb{R}_{\geq 0}$ , is generated around the exploration vehicle. In order to approximate the field variance gradient surrounding the exploration vehicle, a small step size value for  $r$  is chosen. In this thesis,  $r = 3$  vesicles is used. The point on the circle surrounding the exploration vehicle with the highest Kriging prediction variance is set as the next decision point.

### 3.7 The Range Gradient Ascent Path Planner

The Range Gradient Ascent (RGA) Path Planner is similar to the Gradient Ascent path planner, but it sets the radius of the candidate circle surrounding the exploration vehicle to the value of the range,  $a$ , on the field's variogram, i.e. ( $r = a$ ). The points within the range circle are points that are considered spatially autocorrelated, and therefore predictable to a high degree of confidence. By navigating the exploration vehicle to a point on the edge of spatial autocorrelation, the vehicle is motivated to move to points of higher prediction variance as the planner continues to run.

### 3.8 Other Planners

The five planners introduced will be compared to two planners discussed in the Section 1.1 on Previous Works. The first method, the zig-zag (ZZ) method, directs the exploration vehicle through a predetermined path which spirals through the entirety of the field. The second method, the Greedy Next-Best-View (NBV) [17], attempts to reduce Kriging variance by redirecting an exploration vehicle to the point of highest prediction variance after each sample taken.

#### 3.8.1 Zig-Zag Method

A common approach to exploration and patrolling problems is the use of a zig-zag pattern. The methods introduced will be compared to a zig-zagging approach demonstrated in Nikhil Nigam, et al. *Control and Design of Multiple Unmanned Air Vehicles for a Persistent Surveillance Task* (Part II.C.3, Figure 6, [15]). The method will run a Kriging prediction

and variance calculation on the samples taken using the zig-zag explorer, even though a prediction is not specified in the original work. This is to generate measurable and comparable metrics against the path planners introduced.

The zig-zag exploration method will stop the field exploration process when the exploration vehicle traverses a predefined area to scan,  $A_{scan}$ . If the maximum scan area of a field is  $w \times h$ , and the percent of the field to scan is  $p\%$ , then the method will stop exploring when the area  $A_{scan} = \frac{p}{100}wh$  of the field has been sampled. The spacing between each spiral bound,  $r$ , will be pre-calculated in an effort to allow the zig-zag method to cover as much of the field as possible.

$$r = \frac{100}{p} \quad (3.9)$$

The zig-zag method starts at the first point on the field (upper-left corner), and moves to the waypoint coordinate  $(\lceil \frac{w}{2} - r \rceil, \lceil \frac{h}{2} - r \rceil)$ . Every planner will set the initial waypoint of the exploration vehicle to this point on the field. This is done so the initial set of samples of each of the planners is identical.

### 3.8.2 Greedy Next-Best-View

The Greedy NBV method, demonstrated in [17], triggers the selection of a new waypoint after every new sample is taken by the exploration vehicle. The point that is chosen is the point on the field with the highest Kriging prediction variance at the time of point selection. After the next sample is taken on the way to the point of highest variance, the planner recomputes the field's variogram model, Kriging prediction, and field variance to recalculate the next decision point. The Greedy NBV planner will initiate by conducting



an initial sweep on the main diagonal of the field and stop at the middle point of the field. This is done in order to collect a set of samples to make an initial decision.

### 3.9 Planner Discussion

The Monte Carlo path planner takes into account a set of noisy trajectories, and compares their estimated return on investment similarly to the  $N$ -HV method. Given enough trajectories, as  $N$  gets larger for  $N$ -HV and MCPP and  $M_{mc}$  gets larger for MCPP, the planners could find a path that will reduce overall field uncertainty in a more brute-force way over the HV, GA, and RGA methods introduced. The disadvantages of MCPP and  $N$ -HV lie in the fact that the cost of each next move taken is not considered directly because an entire trajectory is considered, in its entirety, at once. A more optimal approach to this planner would be to take into consideration the cost of each waypoint selected on the trajectory, and amend only the best waypoints found along the way. Furthermore, the MCPP approach is more computationally expensive (for  $N > 1$ ,  $M_{mc} > 1$ ) when compared to HV and  $N$ -HV because of the need to re-predict the field for each candidate trajectory calculated. The GA method becomes more computationally expensive as more samples are taken on the field, as this makes it more difficult to recompute the variogram model and Kriging predictions for the field at each step of the path. The HV, GA, and RGA planners are the simplest to compute because they only require one run of the Kriging field predictions and variance calculations, followed by a simple search for the highest field variances from a specified set of points.

The Greedy Next-Best-View planner, though similar to the GA and RGA planners in-

roduced, does not purposefully attempt to directly sample the point of highest prediction variance from a set of candidate points. Both the GA and RGA planners arrive at a point of high variance from a set of candidate points, and sample it, where as the Greedy NBV will only move in the direction of the point of highest variance. As the field sizes get larger, the point of highest variance could be relatively distant from the point that is actually explored using Greedy NBV. The Greedy NBV method is therefore not expected to do well for larger fields. This is because the exploration vehicle can run into the problem where it will stick in a low variance valley. Once trapped in the valley, the vehicle will repeatedly move between two previously explored points until the end condition is met. The method will only be compared to the introduced methods in this thesis using course as presented in [17].

A preplanned method, like the zig-zag method, does not intentionally attempt to suppress overall field prediction uncertainty, and because of this, may not predict the states of interest of the field to a high enough degree of accuracy when compared to the direct variance suppression methods. The zig-zag method does however attempt to sample an even distribution of path across the target field, allowing the method to estimate the field states to a high degree of accuracy.

## Chapter 4

# Simulation Framework

The methods described in Chapter 3 on Path Planning were implemented within a simulation environment in *MATLAB* in order to show the effectiveness of the introduced path planners. The target fields in the simulations are represented as randomly generated matrices of a desired size. The autocorrelation factor of the field in the simulation is an adjustable factor that determines the likeliness of each pair of neighboring points as a function of distance.

For a given waypoint destination, a simulated vehicle will pass over every vesicle on the line connecting its original position and its final position. The vehicle's heading angle can be controlled at any point in the simulation. The path planner directly feeds a control heading angle to the simulated vehicle. A trajectory (a set of waypoints),  $T$ , calculated in Chapter 3, is loaded into a waypoint queue for the vehicle, where each waypoint is met one after another. After meeting the final waypoint in the trajectory, the next set of waypoints is calculated by a path planner. The process continues until the termination condition

(maximum area scanned) is satisfied. A sample is taken at every possible vesicle that the vehicle passes over. The location and value of each sample is stored in the vehicle object's memory for later use in the prediction procedures.

## 4.1 Generating a Target Field

The simulation yields a target field that is of variable height  $h$ , and width  $w$ . Each vesicle in the field is exactly the area of the sensor footprint of the simulated vehicle's sensor. This is to make the sensor measurements as ideal as possible, so no samples are missed when a vesicle is scanned.

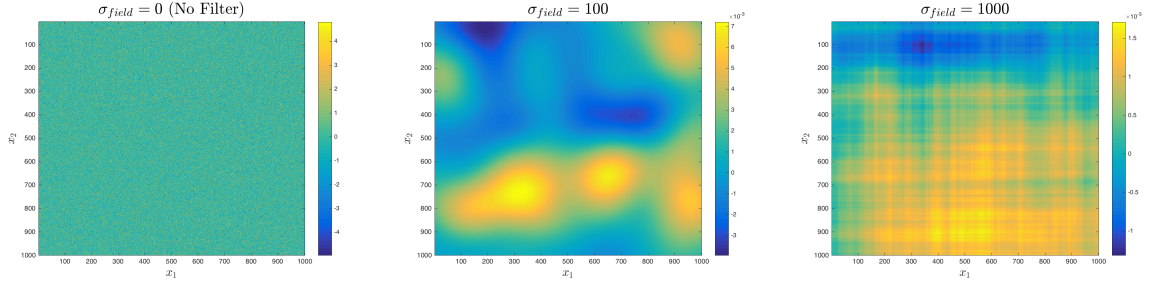
The field is composed of a single feature which is autocorrelated spatially. Initially, the points on the field are generated from a normal distribution with a standard deviation of 1, and expected value of 0. The field is then convolved with a two dimensional Gaussian filter,  $G(x, y, \sigma_{field})$  (Equation 4.1), with a variable standard deviation,  $\sigma_{field} \in \mathbb{R}_{\geq 0}$  which sets the radius of the filter.

$$G(x, y, \sigma_{field}) = \frac{1}{2\pi\sigma_{field}^2} e^{-\frac{x^2+y^2}{2\sigma_{field}^2}} \quad (4.1)$$

where  $x \in \mathbb{N}$ ,  $y \in \mathbb{N}$ , and for all values  $x \in [1, w]$ ,  $y \in [1, h]$ .

The Gaussian filter “smooths” the field in order to simulate autocorrelation. In *MATLAB*, the *imfilter* function is used to perform a 2D convolution on the randomly generated field. The result is a randomly-generated, variably-sized, and autocorrelated field with a unit-less feature of interest. One such field can be observed in Figure 2.1a. As the value of the standard deviation of the Gaussian filter kernel,  $\sigma_{field}$ , increases, the field exhibits higher spatial autocorrelation. Inversely, when  $\sigma_{field}$  is close to zero, the field becomes has

no signs of spatial autocorrelation.



(a)  $\sigma_{field} = 0$ . The field exhibits no spatial autocorrelation. (b)  $\sigma_{field} = \frac{w}{10} = 100$ . The field appears to exhibit some degree of spatial autocorrelation. (c)  $\sigma_{field} = w = 1000$ . The field exhibits a high degree of spatial autocorrelation.

Figure 4.1: A field is generated using a random number generator with a zero mean normal distribution with variance 1. Varying degrees of spatial autocorrelation are shown for different values of  $\sigma_{field}$ .

## 4.2 Simulation Environment

The simulation, once started, runs a single exploration method at a time starting with the same random number seed. A generated field is initially unknown to a vehicle object, but samples are collected as it passes along the field. The variances of Kriging predictions, the currently predicted field, and the path traversed are plotted along with the actual field being explored.

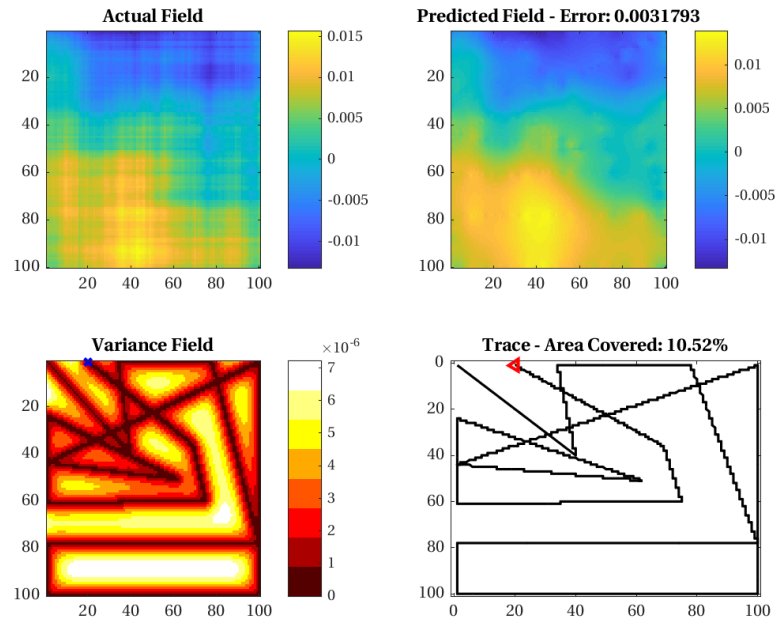


Figure 4.2: Highest Variance Path Planning. The actual field (top left), predicted field (with error) (top right), variance field (bottom left), and traversed path (bottom right).

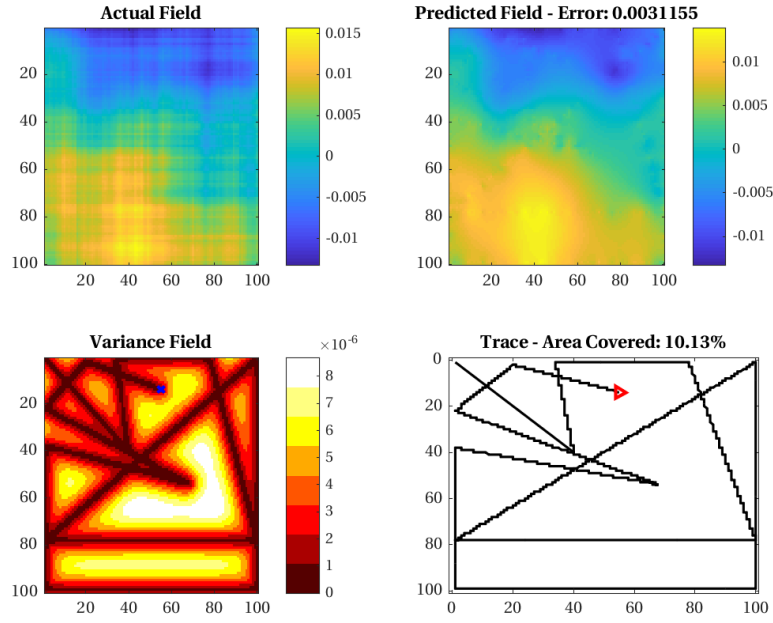


Figure 4.3:  $N$  Highest Variance Path Planning ( $N = 5$ ). The actual field (top left), predicted field (with error) (top right), variance field (bottom left), and traversed path (bottom right).

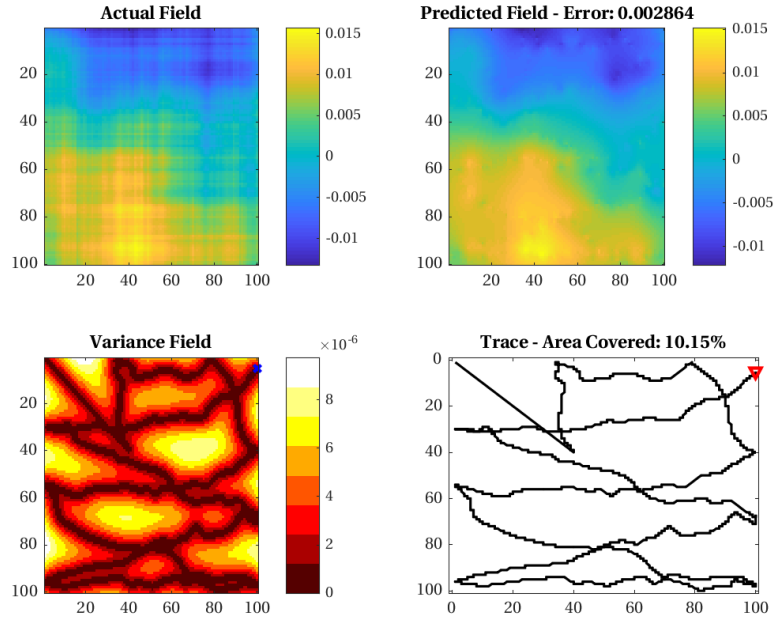


Figure 4.4: Monte Carlo Path Planning ( $N = 5$ ,  $M_{mc} = 5$ ). The actual field (top left), predicted field (with error) (top right), variance field (bottom left), and traversed path (bottom right).



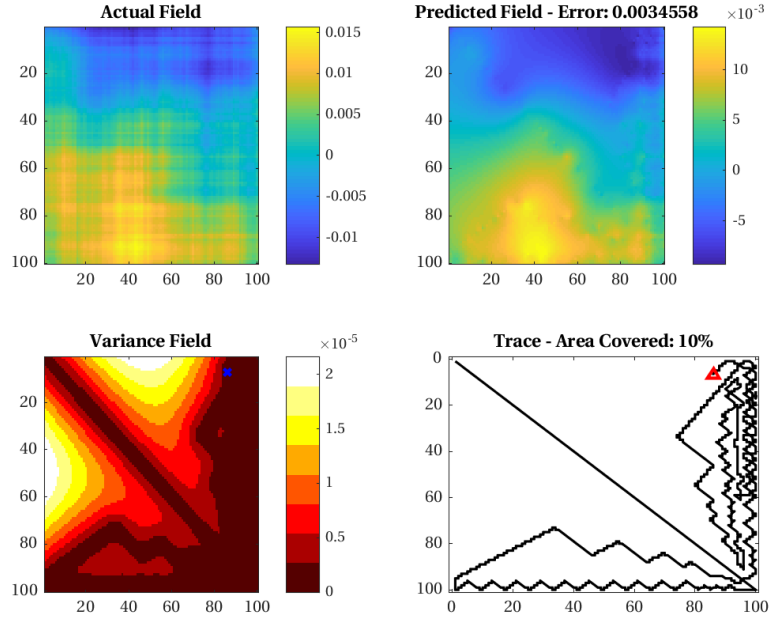


Figure 4.5: Gradient Ascent Path Planner. The actual field (top left), predicted field (with error) (top right), variance field (bottom left), and traversed path (bottom right).

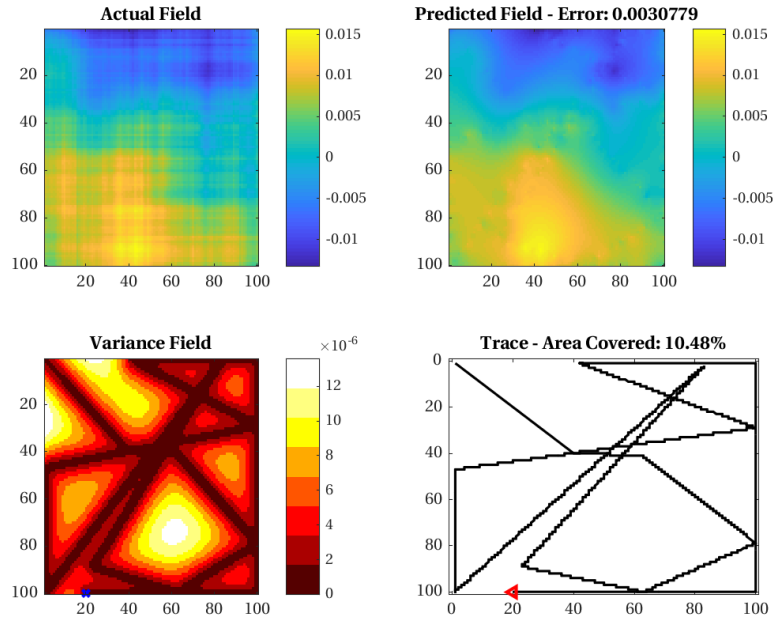


Figure 4.6: Range Gradient Ascent Path Planner. The actual field (top left), predicted field (with error) (top right), variance field (bottom left), and traversed path (bottom right).

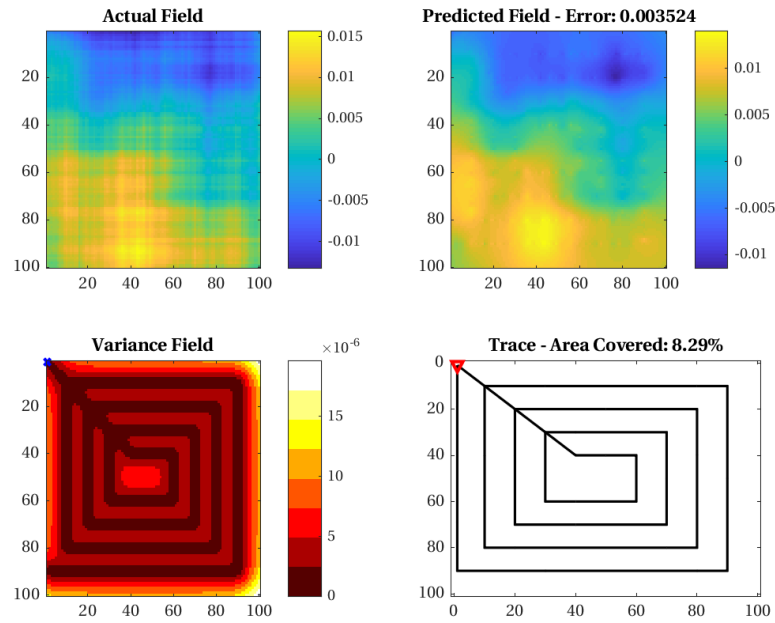


Figure 4.7: Zig-Zag Method. The actual field (top left), predicted field (with error) (top right), variance field (bottom left), and traversed path (bottom right).

## Chapter 5

# Results

The five path planners: HV,  $N$ -HV, and MCPP, GA, and RGA, introduced in Chapter 3, all aim to reduce the overall prediction uncertainty of a target field given a limited amount of exploration time. They accomplish the task by calculating variances of a target field's predictions and attempting to choose a trajectory that reduces overall uncertainty.

The number of trajectories compared in both the  $N$ -HV and MCPP methods,  $N$ , is set to  $N = 5$  for the simulated results. For the MCPP method, an additional  $M_{mc} = 5$  Monte Carlo trajectories are calculated for each of the  $N$  trajectories. The target field size of the fields compared in the simulation have unit-less vesicle dimensions of  $100 \times 100$ . A random number generator seed of 2 is used to generate a set of runs in an effort to show the methods for a variety of random fields and trajectories for MCPP. Appendix A contains the same runs for a different random seed to assess variability. The autocorrelation factors of the field will be varied in an effort to show the effectiveness of the methods for different field statistics.

The prediction errors and variances of the simulation results are normalized to an a priori variance. All methods shown, except for the 10% and 20% scan zig-zag methods, begin by exploring and sampling along the main diagonal of the field down to a common point. The point,  $(\lceil \frac{w}{2} - r \rceil, \lceil \frac{w}{2} - r \rceil)$ , is the initial waypoint for a 30% scan limited zig-zag method, where  $r$  is the zig-zag radius defined in Equation 3.9. This is done so that the initial set of samples of each of the planners is identical. The points sampled along the common initial trajectory are used to run a Kriging prediction and variance calculation for the field. The overall field confidence for the predicted field from the common samples will be the a priori variance normalization value,  $\sigma_{ap}^2$ , for that field. The prediction errors will be normalized to  $\sqrt{\sigma_{ap}^2}$ .

The zig-zag method will be run on varying fields. The preplanned zig-zag method's path will be constant for any field of a given size. For a  $100 \times 100$  field, the zig-zag method will produce the paths in Figure 5.1 for a near 10%, 20%, and 30% scan limit.

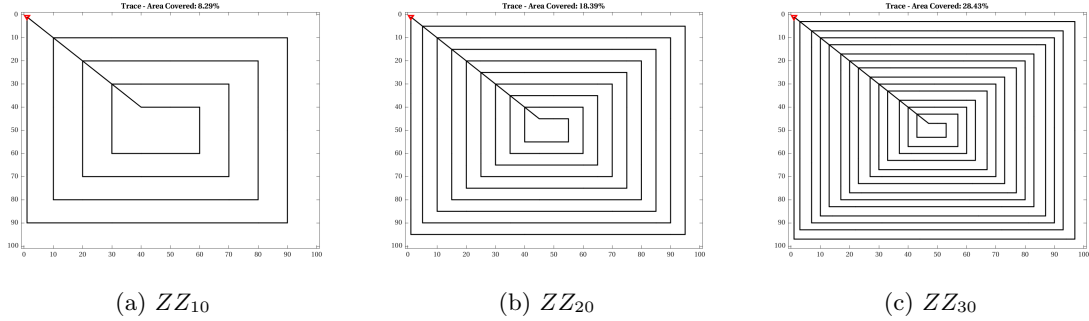


Figure 5.1: Exploration of a field of size  $100 \times 100$  using the zig-zag method for three different percentage scan limits.

## 5.1 Prediction Error Calculation

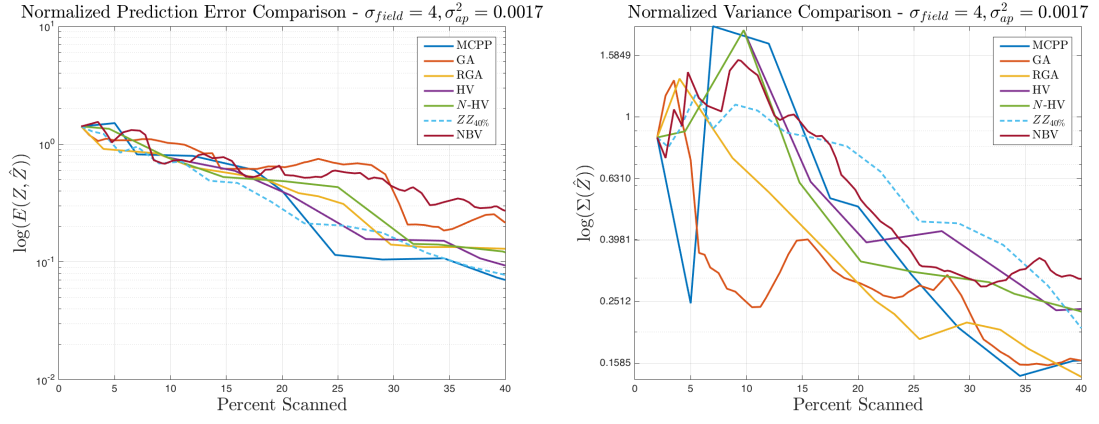
The quality of each path planner will be judged by its ability to explore a field with a fixed exploration path length. The prediction error of each method will be used as a criterion of path planning quality.

The prediction error function,  $E(Z, \hat{Z})$ , will be the average root mean square (RMS) error for all  $h \times w$  points on the actual field,  $Z$ , and the predicted field,  $\hat{Z}$ .

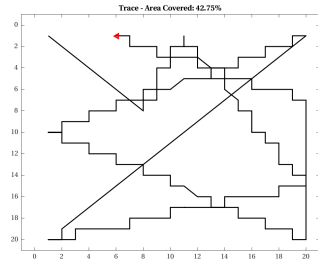
$$E(Z, \hat{Z}) = \frac{1}{hw} \sum_{\forall \mathbf{s}_i \in Z} (Z(\mathbf{s}_i) - \hat{Z}(\mathbf{s}_i))^2 \quad (5.1)$$

## 5.2 Comparing to Greedy Next-Best-View

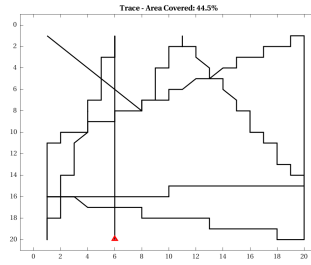
The majority of results are run on a field of  $100 \times 100$  vesicles. Since the Greedy Next-Best-View method fails on a large number of vesicles, a reduced field of  $20 \times 20$ , with an autocorrelation factor,  $\sigma_{field}$ , equal to 4, was used for this comparison. The variogram range value for the field generated is equal to approximately 10 vesicles, which is approximately 5 times more autocorrelated than the field of size  $15 \times 18$  shown in [17]. Each of the five introduced path planners, along with the zig-zag and Greedy NBV methods are compared against each other limited to a 40% scan.



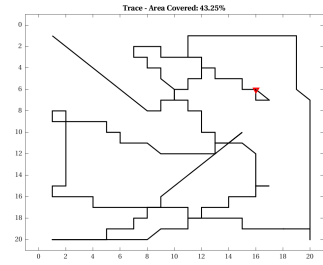
(a) Normalized prediction errors for each method. (b) Normalized prediction variances for each method.  
Figure 5.2: Prediction error and variances for an exploration of a field of size  $20 \times 20$ ,  $\sigma_{field} = 4$ , random seed 2.



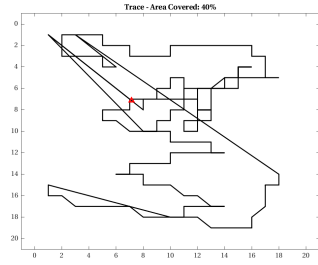
(a) Highest Variance



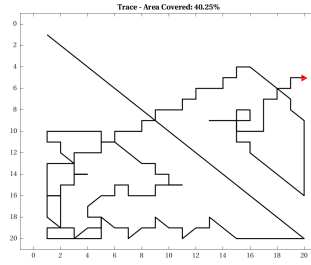
(b)  $N$  Highest Variance



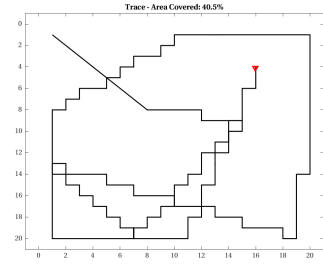
(c) Monte Carlo



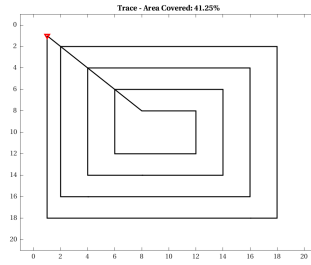
(d) Greedy NBV



(e) Gradient Ascent



(f) Range Gradient Ascent

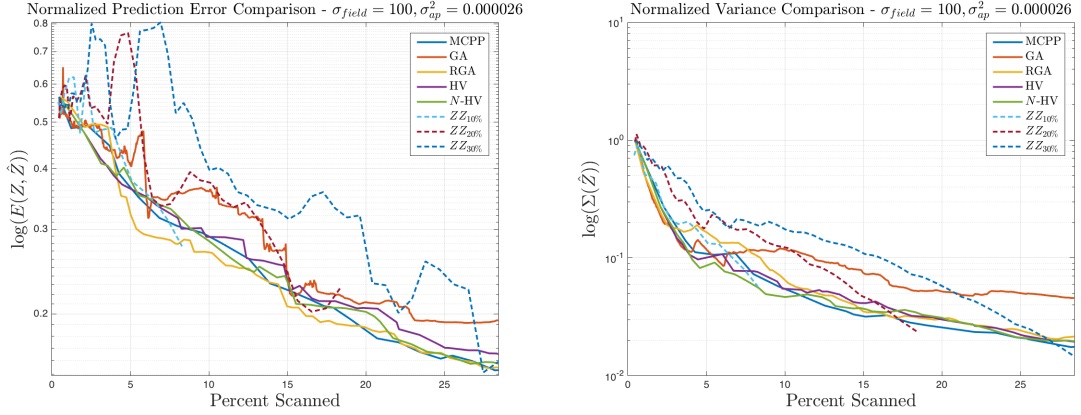


(g)  $ZZ_{40}$

Figure 5.3: A 40% scan limited exploration of a field of size  $20 \times 20$ ,  $\sigma_{field} = 4$ , random seed 2.

### 5.3 High Spatial Autocorrelation Results ( $\sigma_{field} = 100$ )

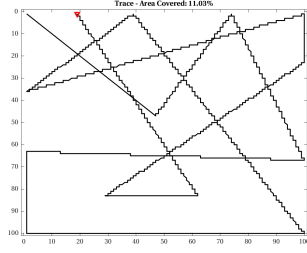
The methods will be compared on target fields generated with an autocorrelation factor,  $\sigma_{field}$ , equal to the field width. A Gaussian filter  $G(x, y, 100)$  (Equation 4.1), is convolved with all points on the field. Paths taken for each of the methods, except for the zig-zag method, are shown for scan areas nearest to the 10%, 20%, and 30% marks in Figure 5.5.



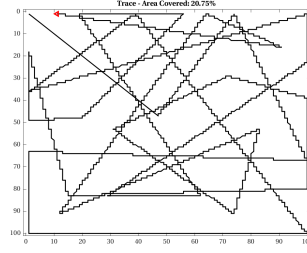
(a) Normalized prediction errors for each method. (b) Normalized prediction variances for each method.

Figure 5.4: Prediction error and variances for an exploration of a field of size  $100 \times 100$ ,  $\sigma_{field} = 100$ , random seed 2.

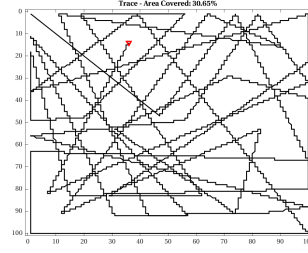




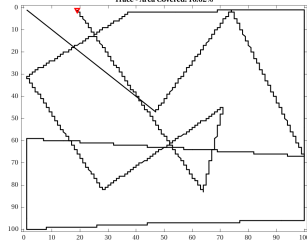
(a) Highest Variance (10%)



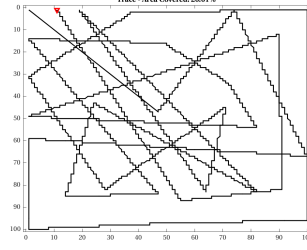
(b) Highest Variance (20%)



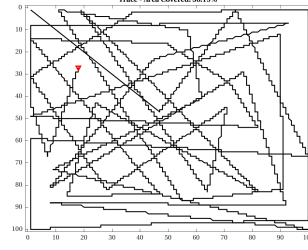
(c) Highest Variance (30%)



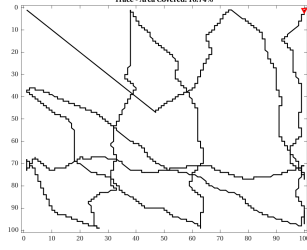
(d) N Highest Variance (10%)



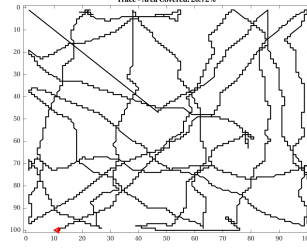
(e) N Highest Variance (20%)



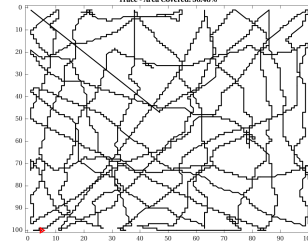
(f) N Highest Variance (30%)



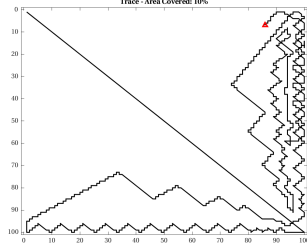
(g) Monte Carlo (10%)



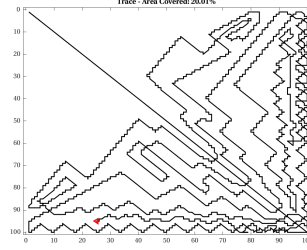
(h) Monte Carlo (20%)



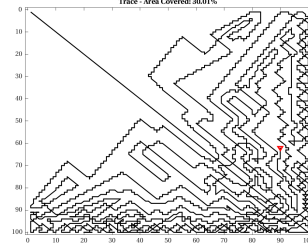
(i) Monte Carlo (30%)



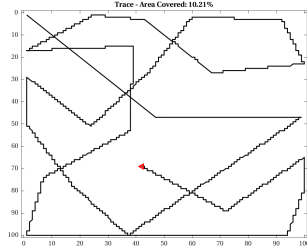
(j) Gradient Ascent (10%)



(k) Gradient Ascent (20%)



(l) Gradient Ascent (30%)



(m) Range Gradient Ascent (10%) (n) Range Gradient Ascent (20%) (o) Range Gradient Ascent (30%)

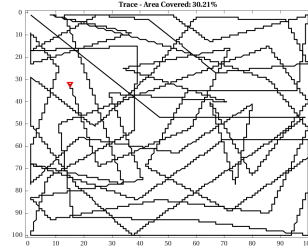
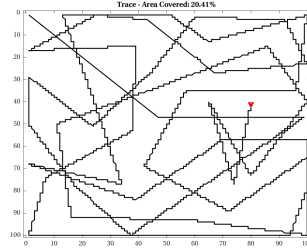
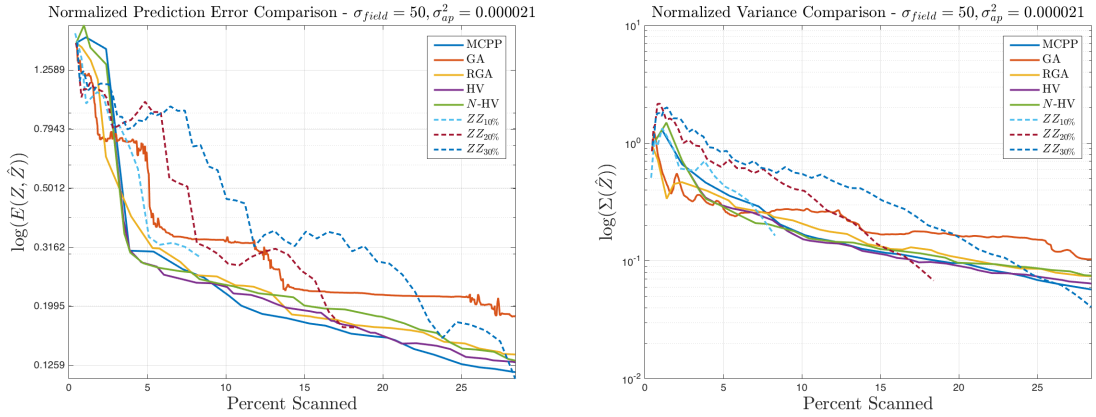


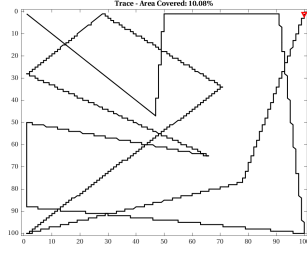
Figure 5.5: Exploration of a field of size  $100 \times 100$ ,  $\sigma_{field} = 100$ , random seed 2.

## 5.4 Half Width Spatial Autocorrelation Results ( $\sigma_{field} = 50$ )

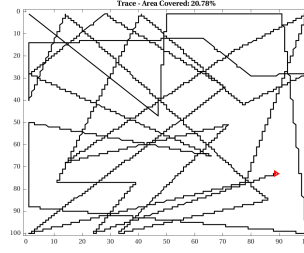
The methods will be compared on target fields generated with an autocorrelation factor,  $\sigma_{field}$ , equal to half of the field width. A Gaussian filter  $G(x, y, 50)$  (Equation 4.1), is convolved with all points on the field. Paths taken for each of the methods, except for the zig-zag method, are shown for scan areas nearest to the 10%, 20%, and 30% marks in Figure 5.7.



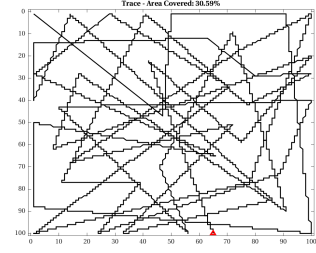
(a) Normalized prediction errors for each method. (b) Normalized prediction variances for each method.  
Figure 5.6: Prediction error and variances for an exploration of a field of size  $100 \times 100$ ,  $\sigma_{field} = 50$ , random seed 2.



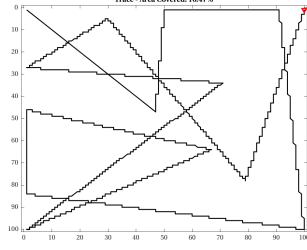
(a) Highest Variance (10%)



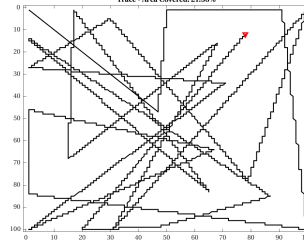
(b) Highest Variance (20%)



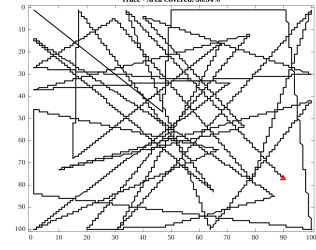
(c) Highest Variance (30%)



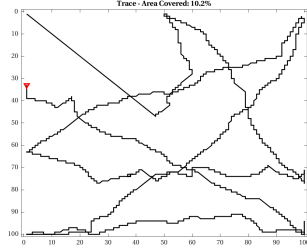
(d) N Highest Variance (10%)



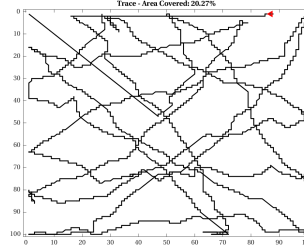
(e) N Highest Variance (20%)



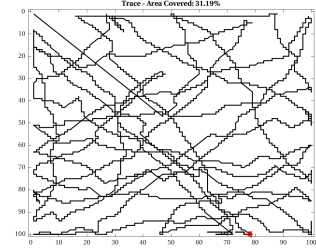
(f) N Highest Variance (30%)



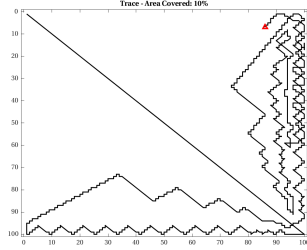
(g) Monte Carlo (10%)



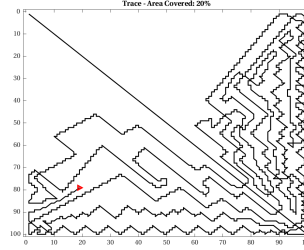
(h) Monte Carlo (20%)



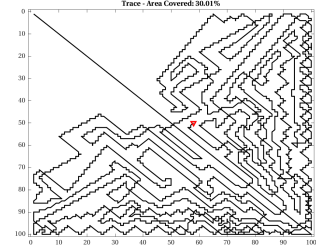
(i) Monte Carlo (30%)



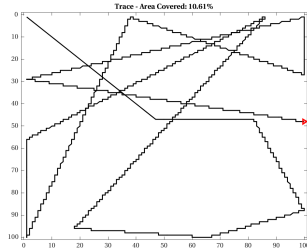
(j) Gradient Ascent (10%)



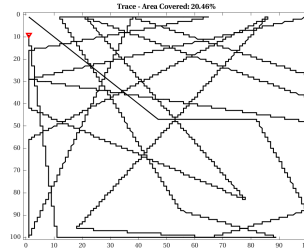
(k) Gradient Ascent (20%)



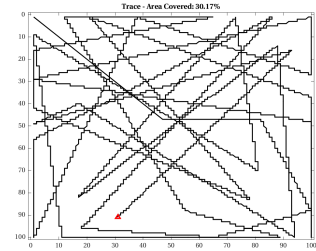
(l) Gradient Ascent (30%)



(m) Range Gradient Ascent (10%)



(n) Range Gradient Ascent (20%)

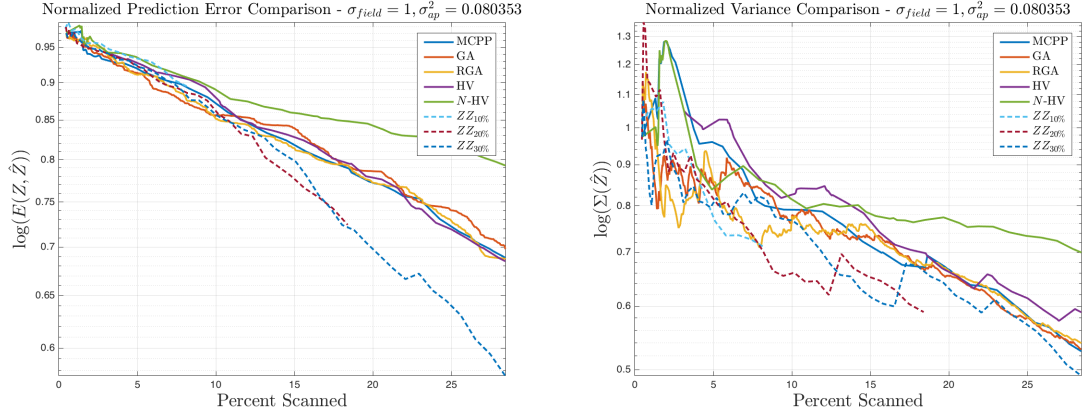


(o) Range Gradient Ascent (30%)

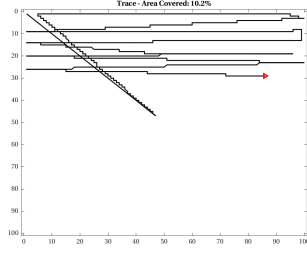
Figure 5.7: Exploration of a field of size  $100 \times 100$ ,  $\sigma_{field} = 50$ , random seed 2.

## 5.5 Low Spatial Autocorrelation Results ( $\sigma_{field} = 1$ )

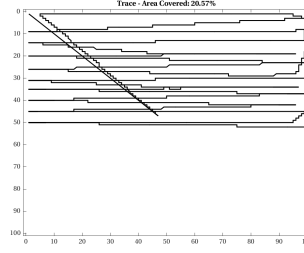
The methods will be compared on target fields generated with an autocorrelation factor,  $\sigma_{field}$ , equal to one. A Gaussian filter  $G(x, y, 1)$  (Equation 4.1), is convolved with all points on the field. Paths taken for each of the methods, except for the zig-zag method, are shown for scan areas nearest to the 10%, 20%, and 30% marks in Figure 5.9.



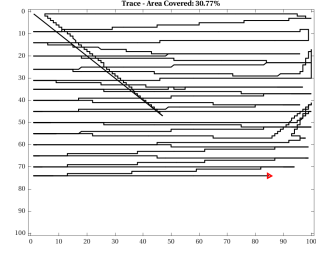
(a) Normalized prediction errors for each method. (b) Normalized prediction variances for each method.  
Figure 5.8: Prediction error and variances for an exploration of a field of size  $100 \times 100$ ,  $\sigma_{field} = 1$ , random seed 2.



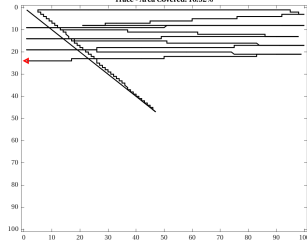
(a) Highest Variance (10%)



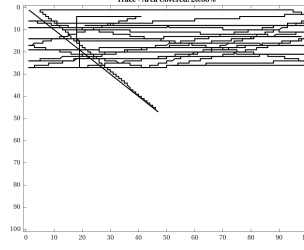
(b) Highest Variance (20%)



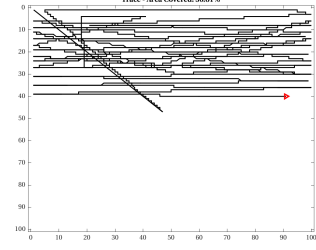
(c) Highest Variance (30%)



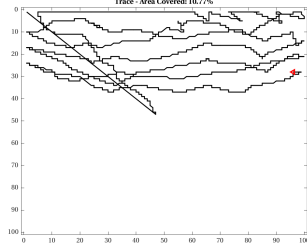
(d) N Highest Variance (10%)



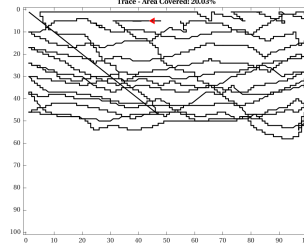
(e) N Highest Variance (20%)



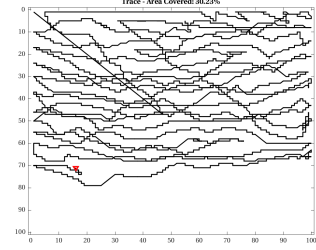
(f) N Highest Variance (30%)



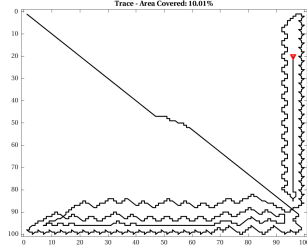
(g) Monte Carlo (10%)



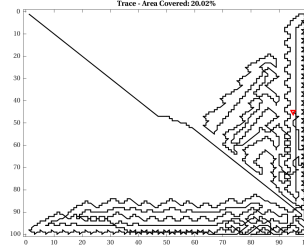
(h) Monte Carlo (20%)



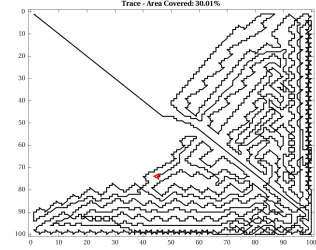
(i) Monte Carlo (30%)



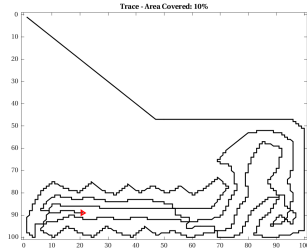
(j) Gradient Ascent (10%)



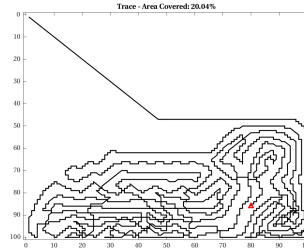
(k) Gradient Ascent (20%)



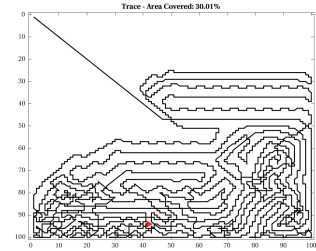
(l) Gradient Ascent (30%)



(m) Range Gradient Ascent (10%)



(n) Range Gradient Ascent (20%)



(o) Range Gradient Ascent (30%)

Figure 5.9: Exploration of a field of size  $100 \times 100$ ,  $\sigma_{field} = 1$ , random seed 2.

## 5.6 Comparing The Methods

While all of the path planners reduce Kriging prediction variance and errors in Section 5.2 and Appendix A.1, the Greedy Next-Best-View method finishes the 40% field scan with the highest prediction error and highest prediction variance. The Range Gradient Ascent, Monte Carlo Path Planner, and Gradient Ascent methods performed the best out of the other methods. This is likely due to the Greedy NBV method's inability to scan points of high variance directly as discussed in Section 3.9. The benefits in Greedy NBV lie in its computational simplicity when compared to the trajectory calculating methods like MCPP and  $N$ -HV. When comparing computational complexity, the HV method selects a point less often than the Greedy NBV method, and when the point is selected, is identical. The GA and RGA methods theoretically run in the same runtime in terms of selection ( $\Theta(hw)$ ). From the simulated results, HV, GA, and RGA would generate better quality mapping over Greedy NBV for a given field with these spatial autocorrelation factors, and would require the same computational ability.

All of the path planning methods reduce prediction variance as they scan more area, for all runs of varying spatial autocorrelation, field, and random seed. The prediction errors drop proportionally to the prediction variance lost over area scanned. The hypothesis that purposefully minimizing Kriging prediction variance in turn reduces prediction error (stated in Section 3.1) has been demonstrated to be true for the results show in this chapter and in Appendix A.

The results show that a field becomes more predictable when it has a larger spatial autocorrelation factor. This is due to the fact that the Kriging method predicts more

accurately if more spatial traits of the field are known and “obvious” to the predictor. When the field is highly predictable (higher values of  $\sigma_{field}$ ), a variance suppressing path planner, like the planners introduced in this thesis, perform better than preplanned paths which do not attempt to directly reduce prediction variance. When comparing the Monte Carlo Path Planner against the zig-zag method for varying autocorrelation factors and varying random seeds (Figures 5.10 and 5.11), the Monte Carlo path planner reduces prediction error and prediction variance to a higher degree for most of the scanning processes. The lower prediction errors associated with MCPP come at the cost of calculating the expected return on a larger number of candidate trajectories ( $NM_{mc}$ ) when compared to any of the other methods.

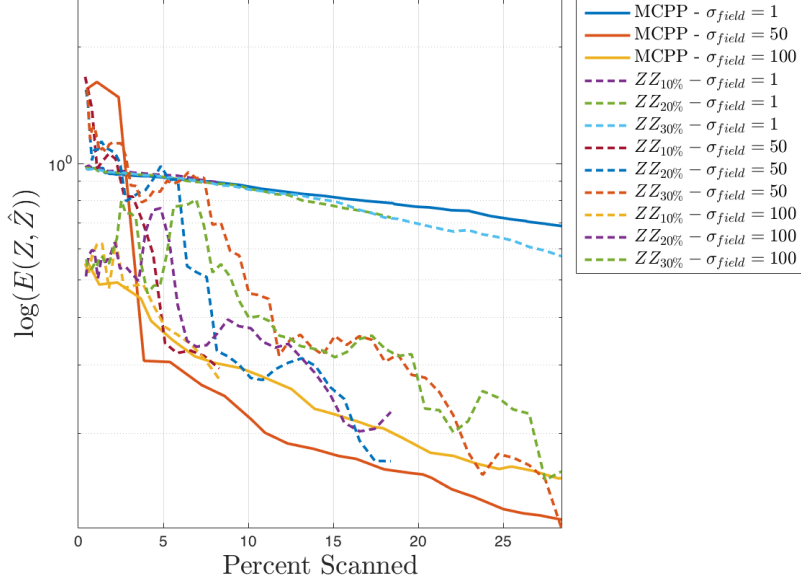
When the autocorrelation factor of the field is low, the preplanned method performs better at reducing prediction error. Unlike the exploration of fields with higher factors of spatial autocorrelation, both the MCPP and  $N$ -HV methods performed poorly for fields with low autocorrelation. This is likely due to feeding back Kriging predictions in the path planning process from poorly predicted points. Furthermore, points near by the exploration vehicle have equally high variances to far away points. The other variance suppressing path planners similarly performed poorly for fields with lower autocorrelations, as there is no guarantee that a field with a low autocorrelation factor will be evenly explored with one of the variance suppressing methods introduced. The differences in the paths taken show that the zig-zag method uniformly scans the field, while the other methods only sample more isolated regions on the field as they become “stuck” scanning in nearby low variance regions until they meet their scan limits. This implies that for lower spatial autocorrelation

in a field, a more evenly distributed sampling regime over the field might reduce prediction error and variance more than directly focusing on reducing prediction variance alone.

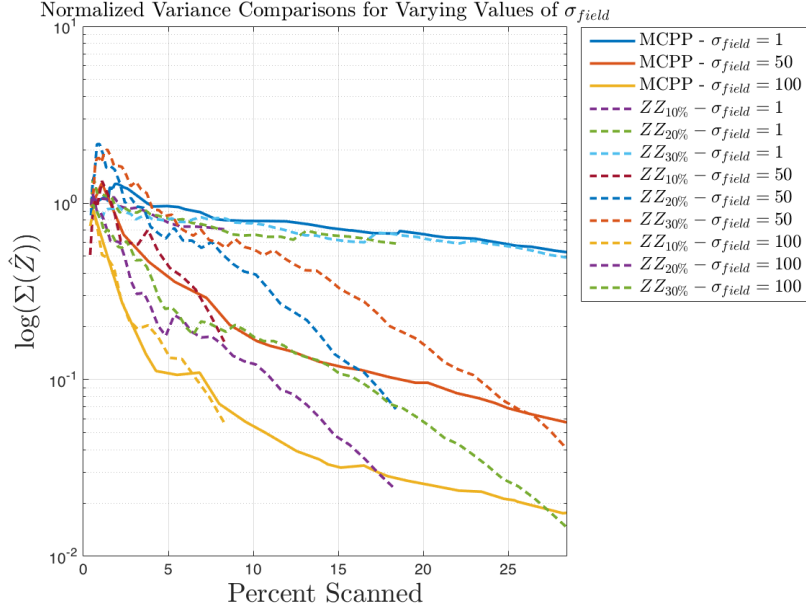
The spatial autocorrelation factor of a target field was shown to be directly proportional to the ability of the non-preplanned planners to explore the field. The proportionality was shown to be true with little variability among different random seeds.



Normalized Prediction Error Comparisons for Varying Values of  $\sigma_{field}$



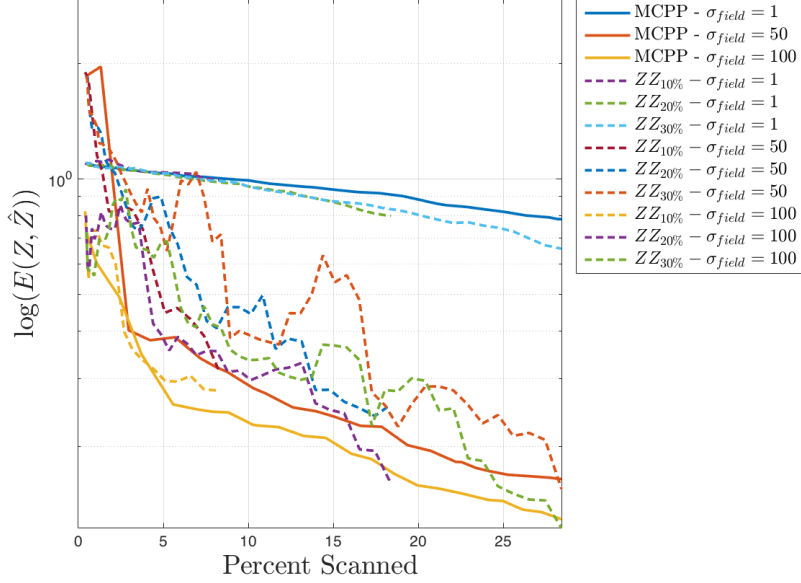
(a) Normalized prediction errors for MCPP and ZZ for  $\sigma_{field} = \{1, 50, 100\}$ .



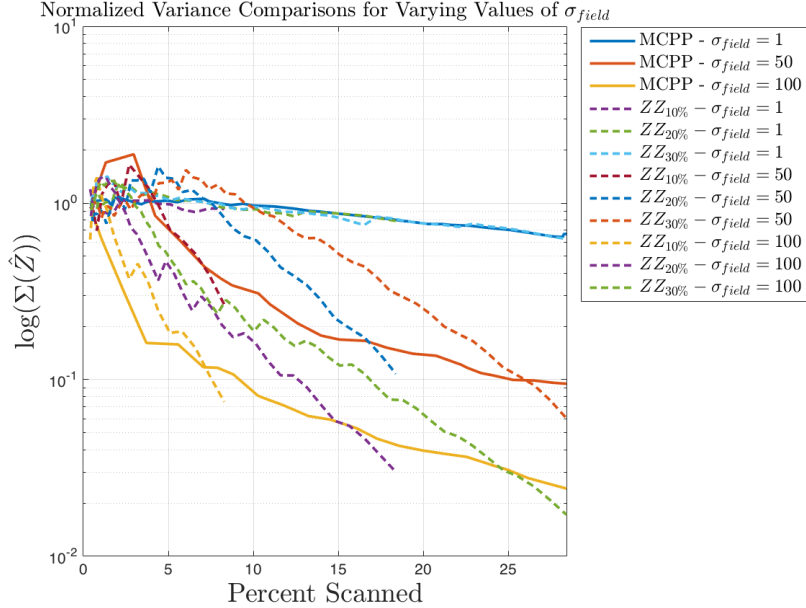
(b) Normalized prediction variances for MCPP and ZZ for  $\sigma_{field} = \{1, 50, 100\}$ .

Figure 5.10: Prediction error and variances for an exploration of 3 different fields of size  $100 \times 100$  for autocorrelation factors of  $\sigma_{field} = 1$ ,  $\sigma_{field} = 50$ ,  $\sigma_{field} = 100$  respectively. Random seed 2.

Normalized Prediction Error Comparisons for Varying Values of  $\sigma_{field}$



(a) Normalized prediction errors for MCPP and ZZ for  $\sigma_{field} = \{1, 50, 100\}$ .



(b) Normalized prediction variances for MCPP and ZZ for  $\sigma_{field} = \{1, 50, 100\}$ .

Figure 5.11: Prediction error and variances for an exploration of 3 different fields of size  $100 \times 100$  for autocorrelation factors of  $\sigma_{field} = 1$ ,  $\sigma_{field} = 50$ ,  $\sigma_{field} = 100$  respectively. Random seed 3.

When compared to the planners introduced, along with the zig-zag method, the Gradient Ascent planner performs the worst in terms of reducing prediction error and variance for size  $100 \times 100$  fields, even with higher autocorrelation factors. This is likely due to the planner limiting its movement to the area directly around it. The planner does not focus on reducing overall field uncertainty globally, but more specifically in the local vicinity of the field, similarly to the Greedy NBV method. The Range Gradient Ascent and Highest Variance planners reduce prediction error more effectively over the standard Gradient Ascent method likely because of their ability to explore and minimize prediction variances more globally. RGA, HV, and  $N$ -HV explore the field by purposefully targeting farther points that are likely to have higher prediction uncertainty by considering points at the edge and outside of the range of autocorrelation on the field. Gradient Ascent was an attempt at hill climbing the variance field. It did not perform well for larger fields because the hills and valleys of the variance field radically change at each decision point in the exploration process. Climbing to the point of highest nearest variance in an effort to maximize variance loss, in an effort to reduce overall field variance, relies on the assumption that the variance field is static. Variance fields were shown to be dynamic, as a function of spatial autocorrelation, as more samples are taken in an exploration process.

## 5.7 Real World Considerations

For an exploration vehicle implementing one or more of the introduced planners on a real system, the computational load required to calculate the Kriging predictions and variances may be a limiting factor. As target field sizes increase, the computational load required

to invert large poorly conditioned matrices may be unrealistic or infeasible. In order to perform a real life exploration, the number of vesicles on the target field must be tuned to meet the requirements (memory and processing power) of the exploration system and minimum exploration specifications.

The initial waypoint in a real exploration mission can be set to any desired point, and is not limited to the point selected in the simulations presented. A consideration to make with regards to setting an initial waypoint is that the variance suppressing path planners require an initial set of samples on the field to make an initial. The first waypoint selected should be at a large enough distance from the starting position so that the planners have enough samples to make a decent initial decision.

Since the variance suppressing planners do not perform well in fields with low levels of spatial autocorrelation, dynamically switching to a preplanned trajectory, like the zig-zag method, might be a useful tactic for exploring a field. The autocorrelation range of the field learned after computing the initial variogram model for the target field can be used to determine whether a preplanned trajectory may yield better results.

## Chapter 6

# Conclusion

The potential in a procedure using the Kriging Method as the core of a field exploration technique with an autonomous vehicle was demonstrated. By characterizing the confidence of the Kriging predictions made from observations in a field, along with uncertainty suppressing motivated path planners, the overall confidence in prediction of a target field as a whole can be maximized without having to scan every point. When compared to an equal path length preplanned exploration trajectory for fields with reasonable spatial autocorrelation factors, the performance criterion, RMS error, was reduced greater by the Kriging variance suppressing path planners introduced. An exploration vehicle could be maneuvered through a field to collect samples in areas of low Kriging prediction confidence. This in turn can increase the quality of prediction of the target field's state of interest to a higher degree of certainty.

For highly spatially autocorrelated fields, with factor  $\sigma_{field} = 100$  (Section 5.3), the Monte Carlo Path Planner (Section 3.5) performed better when compared to the other

path planners demonstrated in terms of reducing field RMS prediction error. The results were closely followed by the  $N$ -HV, RGA, and HV methods, and then the zig-zag method. All methods outperform the 30% zig-zag method up to the 20% scan mark for high to mid autocorrelation factors. For a field with a low spatial autocorrelation factor of  $\sigma_{field} = 1$  (Section 5.5), the preplanned zig-zag methods (20% and 30% scan limited zig-zags) performed the best in terms of reducing prediction error past the 10% scan mark. This is due to the planners ability to scan a more evenly distributed path along the field. A more evenly distributed path across the field implies more spatial characteristics are known about the field, and therefore make the field more predictable.

The zig-zag method outperforms the other methods for fields with very low spatial autocorrelation ( $\sigma_{field} = 1$ ). This is due to the ability of the preplanned zig-zag method to scan the entirety of a target field regardless of its spatial autocorrelation factor. The method will generate a better prediction of the field because it forcibly scans the entirety of the field regardless of any dynamic parameters in the variance field. The other methods rely on the changing variance field to make a waypoint selection. Those planners get stuck in regions of high variance, disallowing exploration in areas not scanned.

## Chapter 7

# Future Work

Future work can be done in an effort to further develop Kriging prediction variance motivated path planning techniques. A comparison of the introduced methods for different vehicle dynamics, e.g. a Dubins Vehicle, can be conducted to show the effectiveness of the introduced methods. Additionally, an implementation of these methods on flying and/or driving hardware can be developed to demonstrate the methods and their differences in a non-simulated setting.

A modification can be made to the Monte Carlo Path Planner and the  $N$ -HV method to create trajectories by amending the best waypoints along the way to a selected decision point. For each waypoint selected, along a leg, the trajectory computed up to a waypoint can be fed back into the prediction and variance calculation process from the predicted points on the trajectory (similar to the current methods for a whole leg). The trajectory, up to that waypoint, can then be compared to a calculated trajectory up to a neighboring waypoint. The trajectory that is considered more optimal up to the candidate waypoint

will qualify to the next phase of waypoint selection. The process will continue until the final intended decision point has been met. This will in turn produce a theoretically more optimal path over the current methods, but at a much higher computational cost.

A method using a combination of the planners introduced and a preplanned trajectory can be done by switching the exploration planning method dynamically based on the autocorrelation range of the field. When the autocorrelation of a region of the field is considered to have low spatial autocorrelation, a preplanned trajectory can be used to explore that section of the field, and another variance suppressing method can be used to explore the regions on the field with higher spatial autocorrelation factors.

Further work can attempt to minimize overall Kriging variance for multiple states of interest across the field while simultaneously predicting more than one state of interest. This can be done by weighing the cost of predicting each of the states of interest dynamically based on the current overall variances of each of the state predictions on the field.

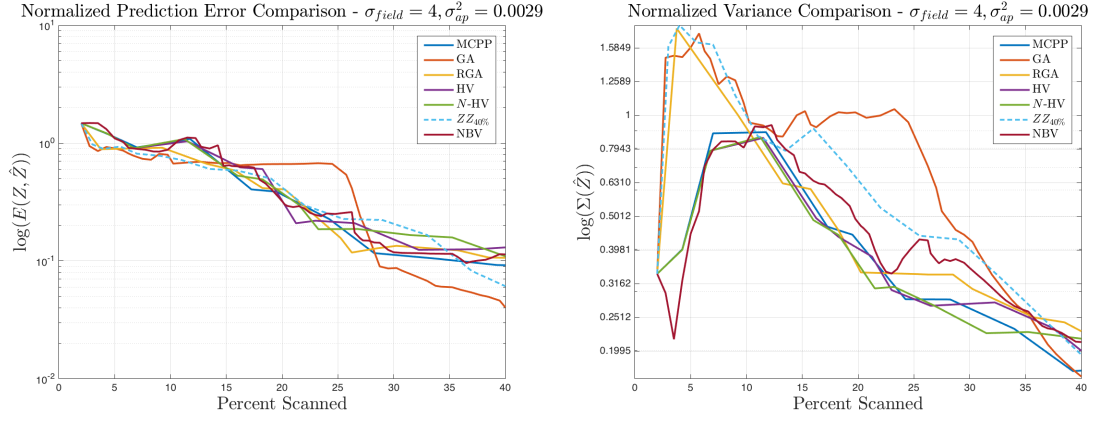


## Appendix A

# Simulation Results with Different Random Seed

The runs in Chapter 5 on Results were performed with the same conditions, but with a different random seed equal to 3.

## A.1 Comparing to Greedy Next-Best-View



(a) Normalized prediction errors for each method. (b) Normalized prediction variances for each method.

Figure A.1: Prediction error and variances for an exploration of a field of size  $20 \times 20$ ,  $\sigma_{field} = 4$ , random seed 3.

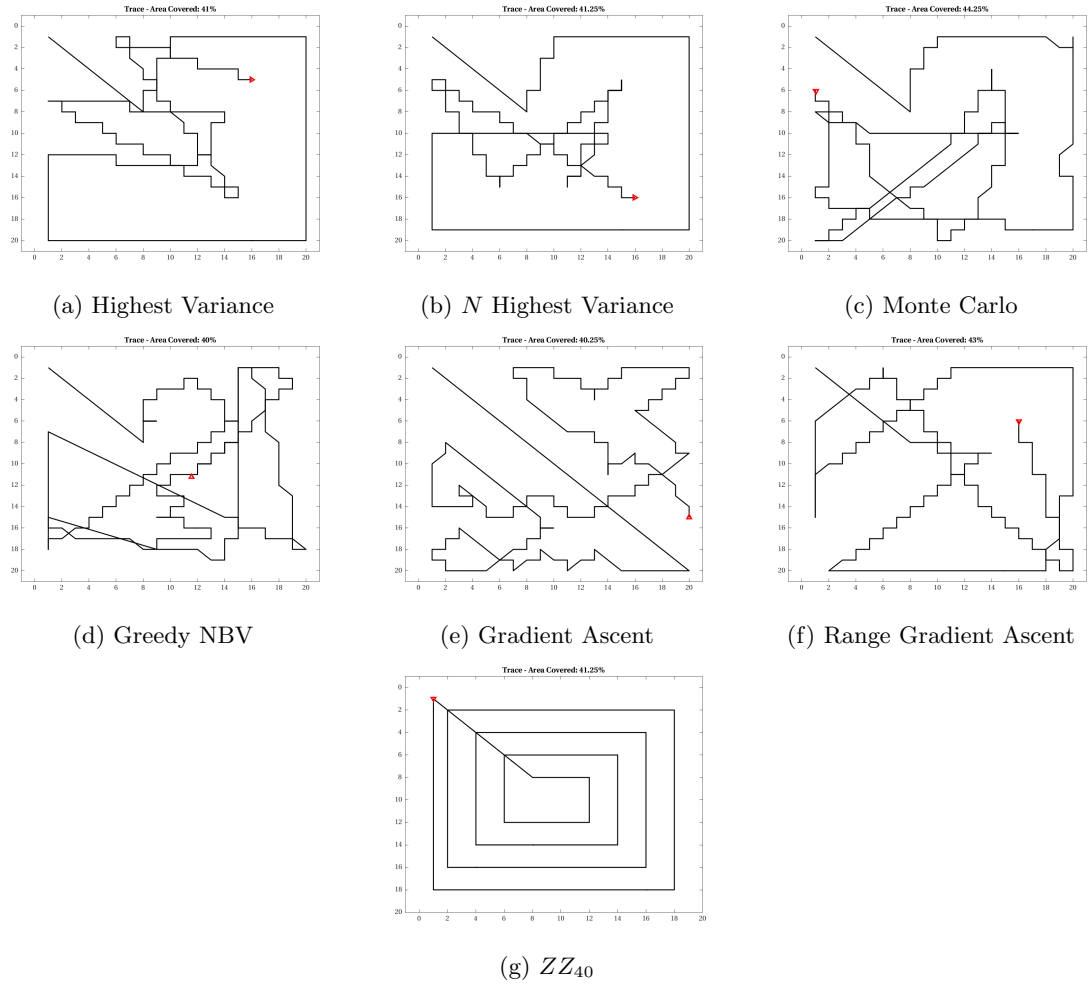
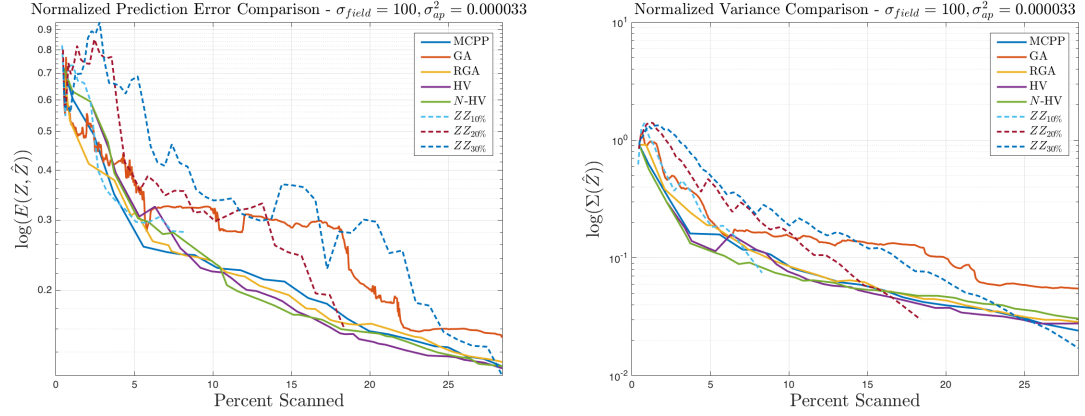
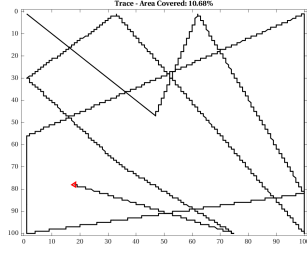


Figure A.2: Exploration of a field of size  $20 \times 20$ ,  $\sigma_{field} = 4$ , random seed 3.

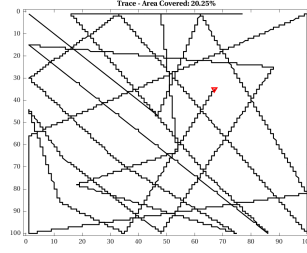
## A.2 High Spatial Autocorrelation Results ( $\sigma_{field} = 100$ )



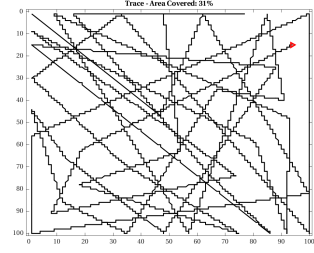
(a) Normalized prediction errors for each method. (b) Normalized prediction variances for each method.  
Figure A.3: Prediction error and variances for an exploration of a field of size  $100 \times 100$ ,  $\sigma_{field} = 100$ , random seed 3.



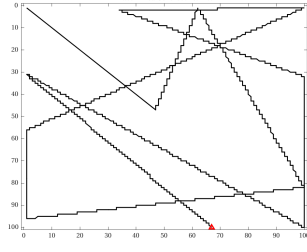
(a) Highest Variance (10%)



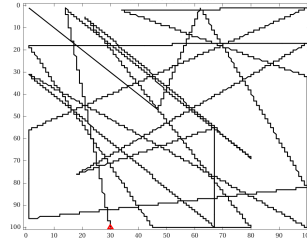
(b) Highest Variance (20%)



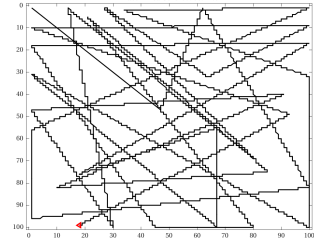
(c) Highest Variance (30%)



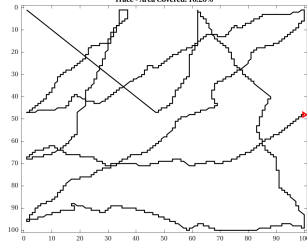
(d)  $N$  Highest Variance (10%)



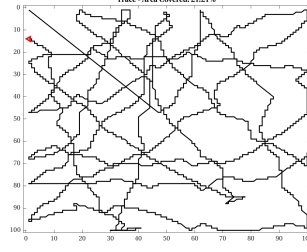
(e)  $N$  Highest Variance (20%)



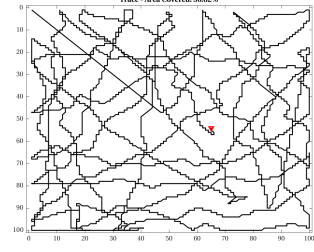
(f)  $N$  Highest Variance (30%)



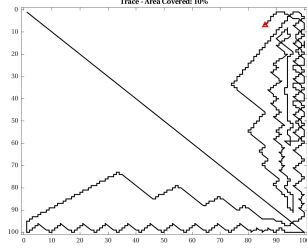
(g) Monte Carlo (10%)



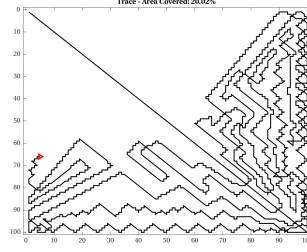
(h) Monte Carlo (20%)



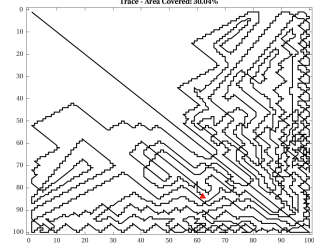
(i) Monte Carlo (30%)



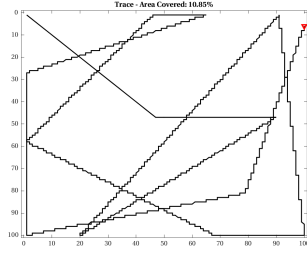
(j) Gradient Ascent (10%)



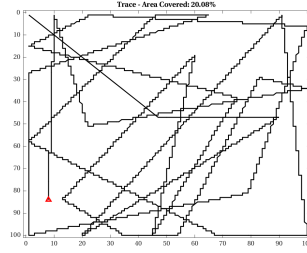
(k) Gradient Ascent (20%)



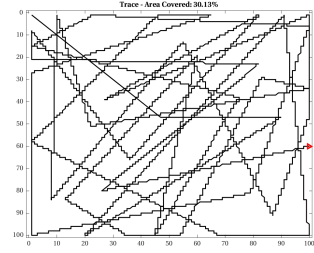
(l) Gradient Ascent (30%)



(m) Range Gradient Ascent (10%)



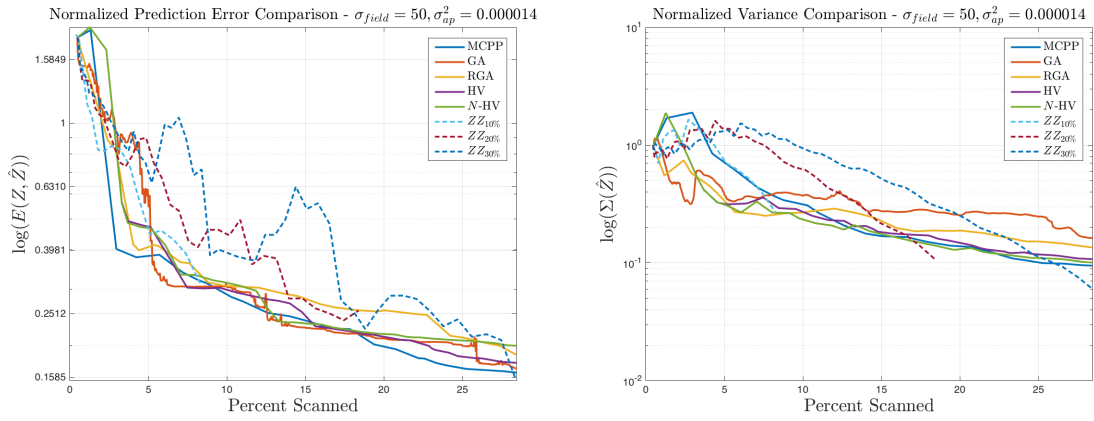
(n) Range Gradient Ascent (20%)



(o) Range Gradient Ascent (30%)

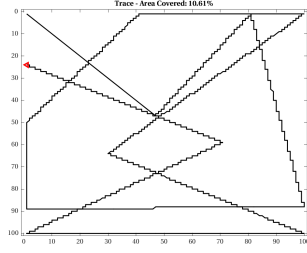
Figure A.4: Exploration of a field of size  $100 \times 100$ ,  $\sigma_{field} = 100$ , random seed 3.

### A.3 Half Width Spatial Autocorrelation Results ( $\sigma_{field} = 50$ )

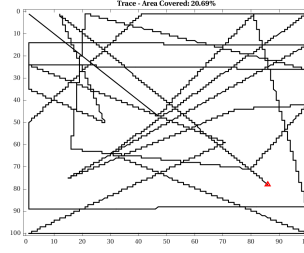


(a) Normalized prediction errors for each method. (b) Normalized prediction variances for each method.

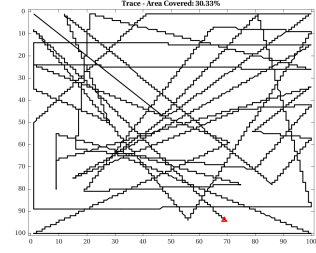
Figure A.5: Prediction error and variances for an exploration of a field of size  $100 \times 100$ ,  $\sigma_{field} = 50$ , random seed 3.



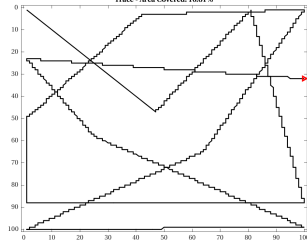
(a) Highest Variance (10%)



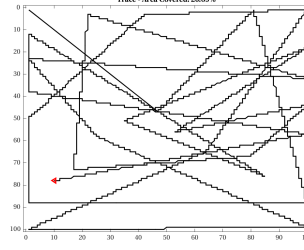
(b) Highest Variance (20%)



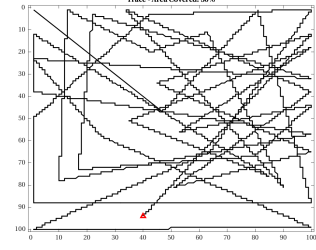
(c) Highest Variance (30%)



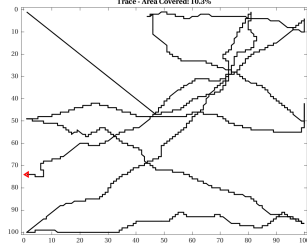
(d)  $N$  Highest Variance (10%)



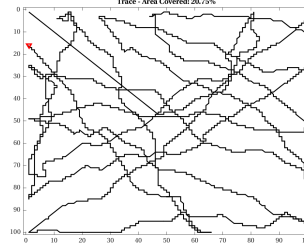
(e)  $N$  Highest Variance (20%)



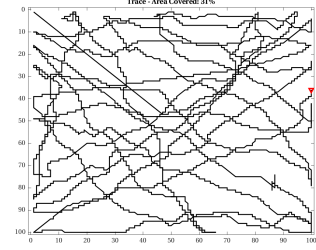
(f)  $N$  Highest Variance (30%)



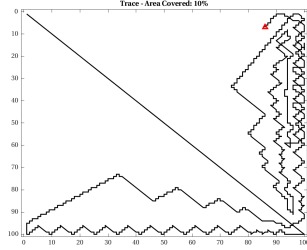
(g) Monte Carlo (10%)



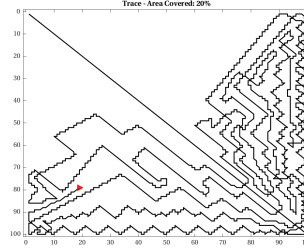
(h) Monte Carlo (20%)



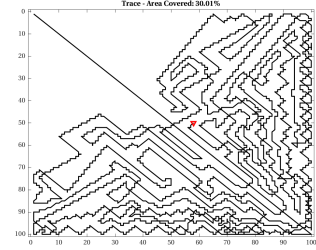
(i) Monte Carlo (30%)



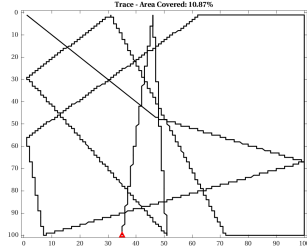
(j) Gradient Ascent (10%)



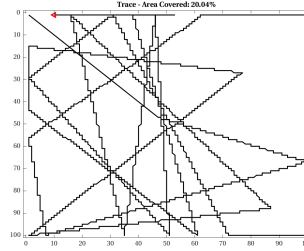
(k) Gradient Ascent (20%)



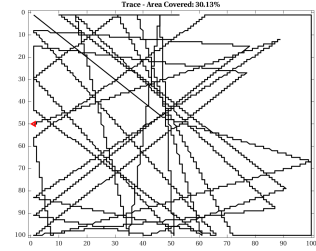
(l) Gradient Ascent (30%)



(m) Range Gradient Ascent (10%)



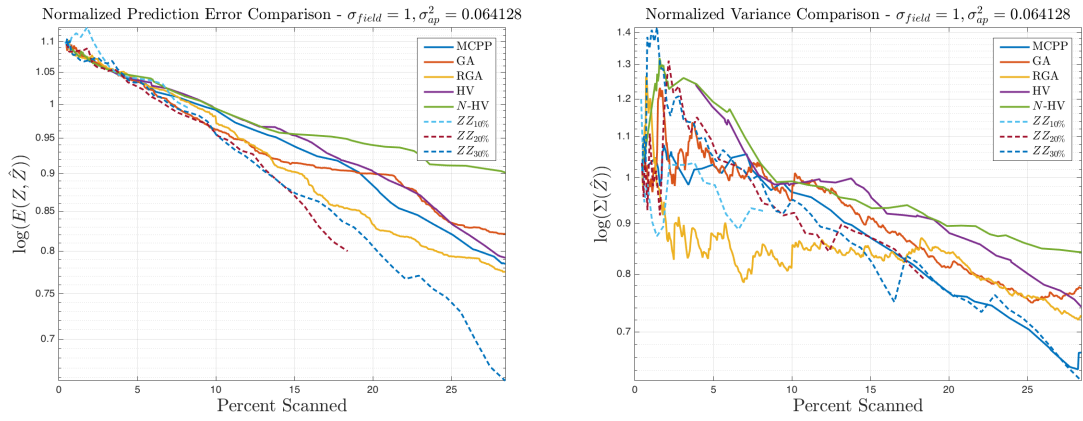
(n) Range Gradient Ascent (20%)



(o) Range Gradient Ascent (30%)

Figure A.6: Exploration of a field of size  $100 \times 100$ ,  $\sigma_{field} = 50$ , random seed 3.

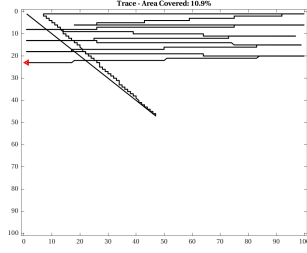
## A.4 Low Spatial Autocorrelation Results ( $\sigma_{field} = 1$ )



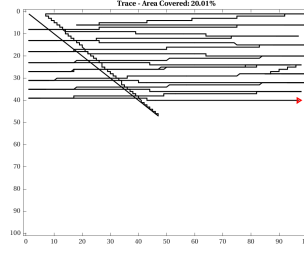
(a) Normalized prediction errors for each method. (b) Normalized prediction variances for each method.

Figure A.7: Prediction error and variances for an exploration of a field of size  $100 \times 100$ ,  $\sigma_{field} = 1$ , random seed 3.

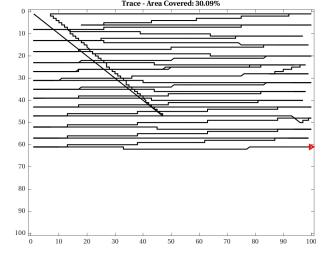




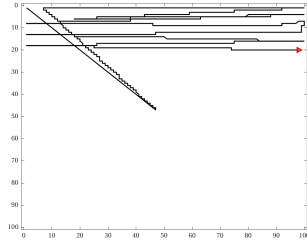
(a) Highest Variance (10%)



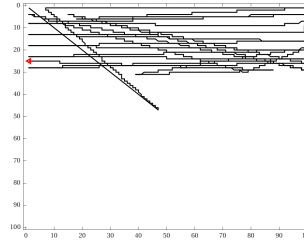
(b) Highest Variance (20%)



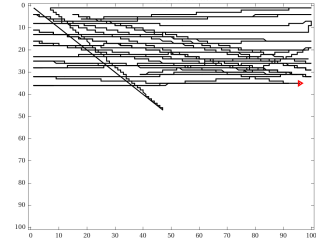
(c) Highest Variance (30%)



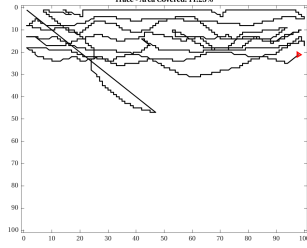
(d) N Highest Variance (10%)



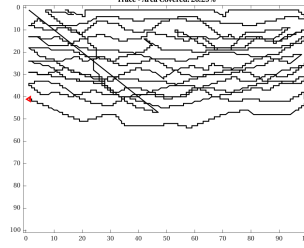
(e) N Highest Variance (20%)



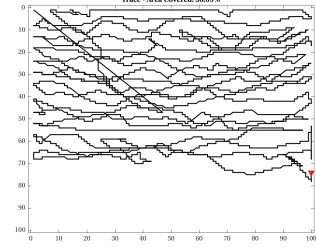
(f) N Highest Variance (30%)



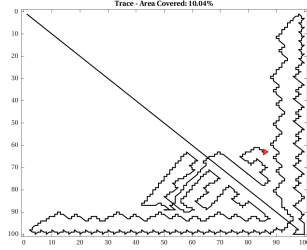
(g) Monte Carlo (10%)



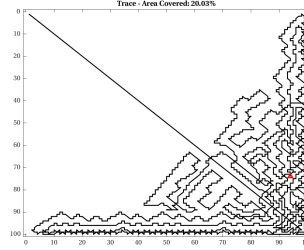
(h) Monte Carlo (20%)



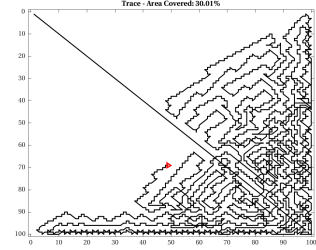
(i) Monte Carlo (30%)



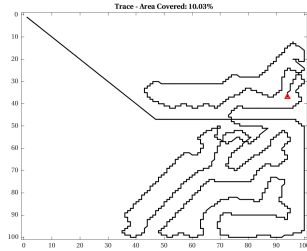
(j) Gradient Ascent (10%)



(k) Gradient Ascent (20%)



(l) Gradient Ascent (30%)



(m) Range Gradient Ascent (10%) (n) Range Gradient Ascent (20%) (o) Range Gradient Ascent (30%)

Figure A.8: Exploration of a field of size  $100 \times 100$ ,  $\sigma_{field} = 1$ , random seed 3.

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