A Procedure for Aerial Field Exploration Using The Universal Kriging Method

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Introduction

The prohibitive costs of building, launching, and maintaining a satellite makes studying and exploring natural phenomena and disasters difficult. It can take up to two days for a desired image to be taken from orbit. Using an Unmanned Aerial Vehicle (UAV) system, a field of interest can be scanned within a more reasonable time frame. Potentially more nuanced data can be gathered from the UAV made observations because of more desirable fields of view and more customizable sensors on-board.

The Kriging Method, a *Best Linear Unbiased Predictor* commonly used in the field of Geostatistics, exploits the statistical properties of natural phenomena to better predict unobserved points from a set of observed points. Using a modified Universal Kriging Method (for "on the fly" use), a prediction and confidence of prediction of the entirety of a given target field can be generated from a set of measurements. From the confidence of predictions calculated by the Kriging Method, a corresponding path-planner for UAV field exploration will be developed to reduce the overall uncertainty of field prediction.

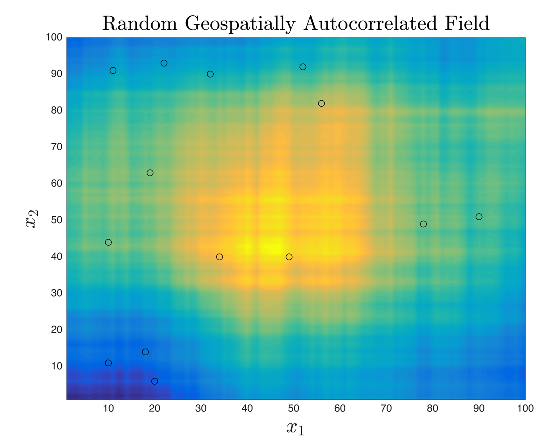
Procedure

- Geospatial Autocorrelation will be assumed for a given target field. From Tobler's First Law of Geography: "Everything is related to everything else, but near things are more related than distant things." [1]
- From a set of observed points, N, on a field, Z, at locations $\vec{s}_i$, a Variogram, $\gamma(h)$, is generated which represents the average covariances between any two points on the target field that are a Euclidean distance, h, away.

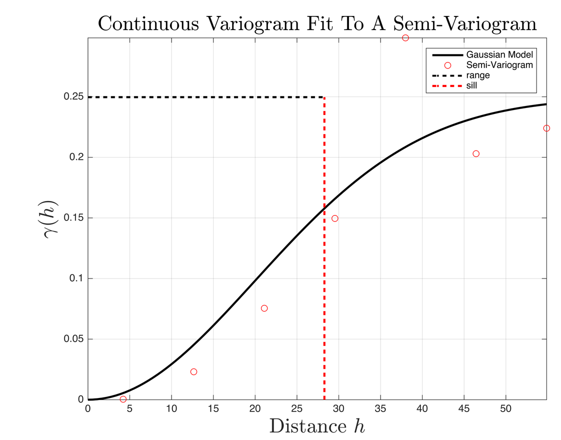
$$h_{i,j} = \|\vec{s}_i - \vec{s}_j\|_2 \tag{1}$$

- A continuous Variogram is fit to a discrete *Semi-Variogram* described by [2]:

$$2\hat{\gamma}(h) := \frac{1}{|N(h_{i,j})|} \sum_{\forall \vec{s}_i, \vec{s}_j \in N(h_{i,j})} |Z(\vec{s}_i) - Z(\vec{s}_j)|^2 \tag{2}$$



a: A geospatially autocorrelated field with samples taken at the locations marked.



b: A Variogram fit from a discrete variogram (Eq. 2).

Figure 1: A randomly generated field is sampled at the marked location in 1a. A Variogram of the observations is made in 1b. The fit Variogram model yields the autocorrelation values between any two points on the target field a distance h away. The sill and range represent where the cut-off point for autocorrelation lies on the target field.

- From the generated Variogram, a Covariance-Variance Matrix, $C \in \mathbb{R}^{|N|\times|N|}$, is generated, where

$$C_{i,j} = Cov(Z(\vec{s}_i), Z(\vec{s}_j)) = \gamma(h_{i,j}) \tag{3}$$

- To predict the value at a location $\vec{s}_0 = [i, j]$, the proximity vector must first be computed.

$$\vec{d}_0 = \begin{bmatrix} C_{1,0} \\ \vdots \\ C_{|N|,0} \end{bmatrix} \tag{4}$$

And then the Kriging Weights are resolved via [3]:

$$\vec{\lambda}_0 = C^{-1}\vec{d}_0 \tag{5}$$

- A Kriging Prediction is made at the point $\vec{s}_0$ by then computing the weighted sum [3]:

$$\hat{Z}(\vec{s}_0) = \sum_{i=1}^{|N|} Z(\vec{s}_i)\lambda_{0i} = [Z(\vec{s}_1)\ldots Z(\vec{s}_{|N|})]\,\vec{\lambda}_0 \tag{6}$$

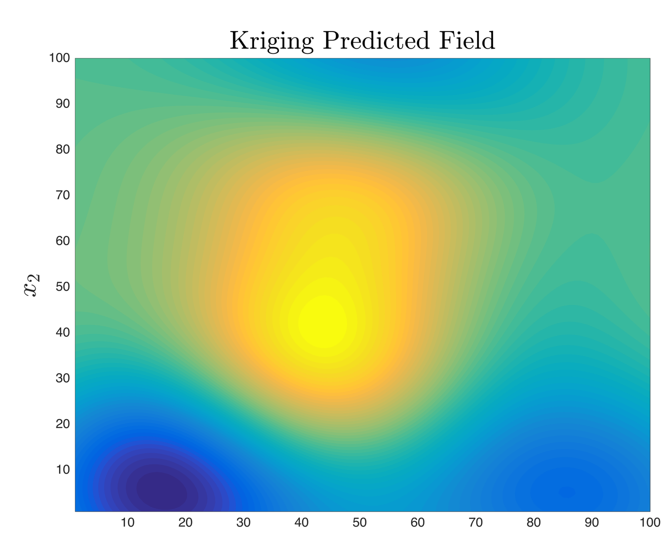
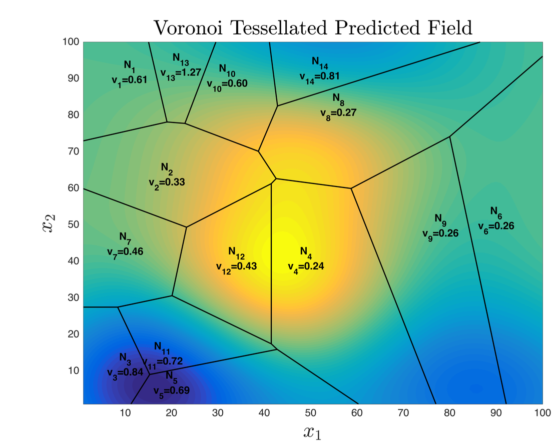


Figure 2: A Kriging Prediction is computed for every point in the target field in Figure 1a.

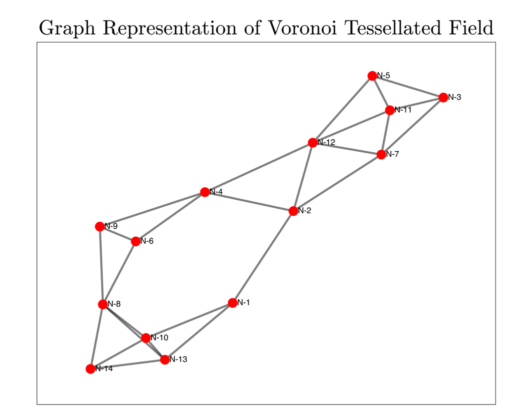
- By tessellating a target field into sub-fields (Voronoi Cells, N_i), a graph representing the average confidence, (or average inverse uncertainty) of prediction, ν_i for each cell (neighborhood i), can be generated from the Kriging method. For a neighborhood with k possible prediction locations:

$$\nu_i = \frac{1}{k}\sum_{j=1}^{k} \langle \vec{d}_j, \vec{d}_j \rangle^{-1} \tag{7}$$

- A graph with vertices representing each neighborhood is constructed. Neighbors adjacent in the tessellated field are adjacent vertices in the graph representation.



a: Voronoi tessellations of the predicted field in Figure 2.



b: A weighted graph representing the adjacencies of the neighbors in Figure 3a

Figure 3: The predicted field is tessellated based on the measured point locations in 3a. An undirected graph is constructed from the tessellations in 3b. The label in each neighborhood identifies the neighborhood name, N_i, and the associated confidence of prediction, ν_i, for that neighborhood.

- For two given vertices in the graph, N_i and N_j, the corresponding edge weight is the sum of the confidences of the two associated neighborhoods:

$$w_{ij} = \nu_i + \nu_j \tag{8}$$

- A path-planning technique based on a shortest-path algorithm is run on the graph. The result will be a path that will guide a UAV into the direction of least confidence while traversing through other uncertain areas. This is because the edge weights are inversely proportional to uncertainty, and maximized in the traversal.
- The path-planning maneuver in turn reduces the uncertainty of prediction of the target field as a whole.
- As more observations are made, the field is re-tessellated and a new graph is created and re-traversed in an attempt to reduce overall uncertainty to a previously specified threshold.

Results

For the k^{th} iteration of tessellation, path-planning, observation, and prediction on the field Z, we will calculate an error using:

$$\eta_{pred}(k) = \sum_{i=1}^{|Z|_k} (Z(\vec{s}_i) - \hat{Z}(\vec{s}_i))^2 \tag{9}$$

Figure 4: Error of prediction over procedure iterations calculated using Eq. 9.

Error decreases as more of the field is observed. This test validates the hypothesis that as more of the field is observed, uncertainty of prediction is suppressed. This implies the effectiveness of the described procedure.

Conclusions

The potential in a procedure using the Kriging Method as an aerial field exploration technique with UAVs was demonstrated. By characterizing the confidence of the Kriging predictions made from observations in a field, along with a natural neighbor selection technique, a method for path-planning was developed to increase overall confidence in prediction of a target field as a whole. Future work to validate these results with a UAV in the loop will be conducted.

References

[1] W. R. Tobler, *A Computer Movie Simulating Urban Growth in the Detroit Region*, Economic Geography, pages 234-240. Clark University, Wiley 1970.

[2] G.Matheron, *Principles of Geostatistics*, Economic Geology, pages 1246-1266. 1963.

[3] Van der Graaf, S.C., *Natural Neighbour Kriging and Its Potential for Quality Mapping and Grid Design*, TU Delft Repository, chapters 1-2. TU Delft 2016.

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